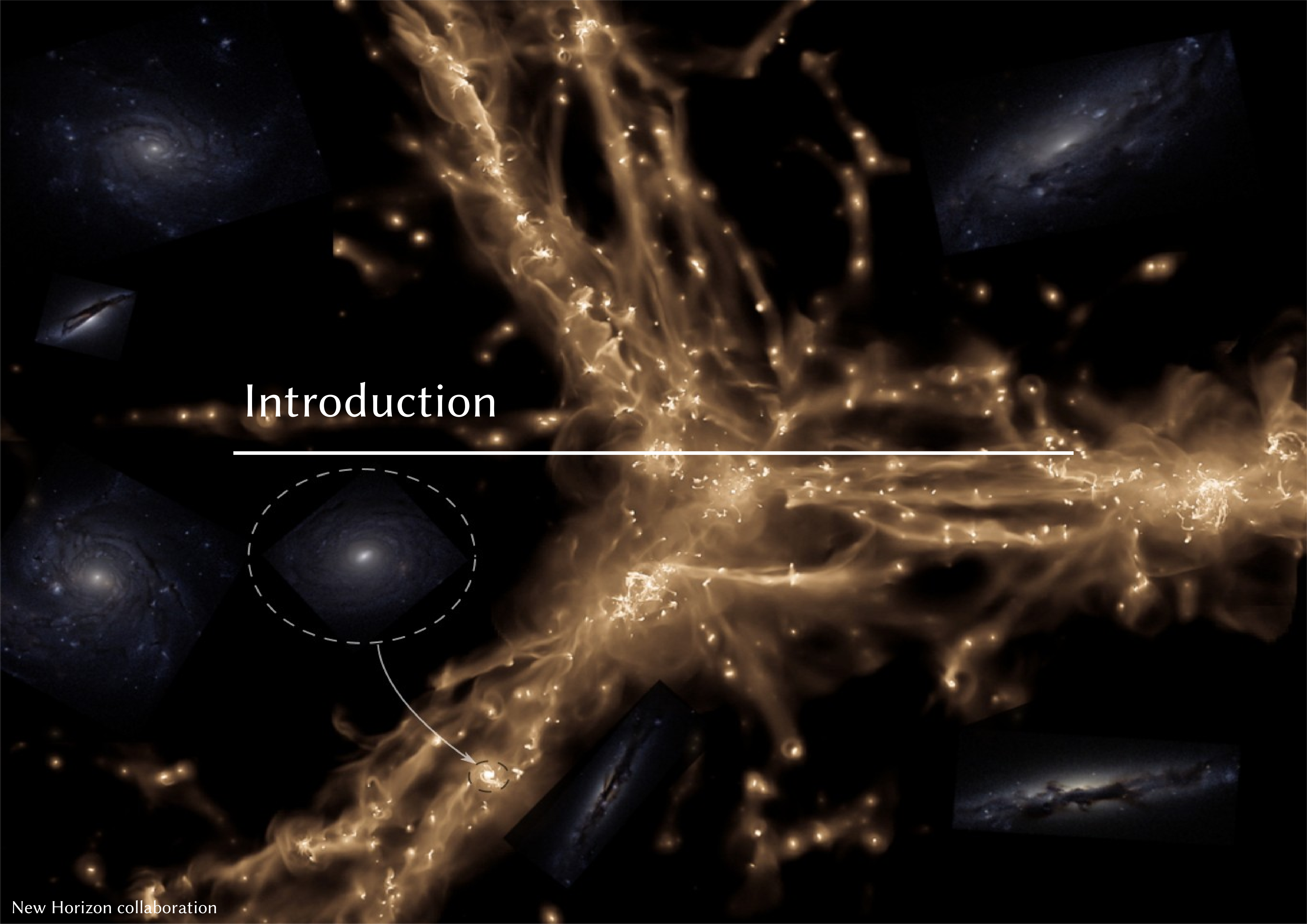
When do cosmic peaks, filaments or walls merge? A theory of critical events in a multi-scale landscape arXiv:2003.04413

University College London

In collaboration with **C. Pichon, S. Codis, Y. Dubois, J.F. Cardoso,** D. Pogosyan, M. Musso, S. Prunet

1 May 2020

Introduction



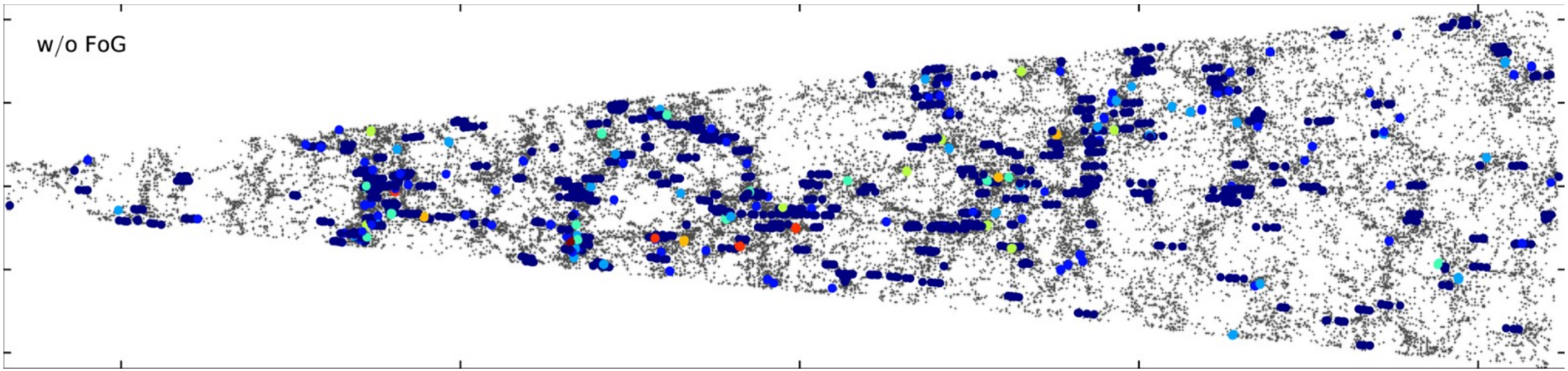
New Horizon collaboration

Motivation(s)

- Unique time in terms of data... DESI, *Euclid*, LSST ⟶ millions of datapoints to play with!
  - Better "halo" model to understand data (White 78, Cooray&Sheth 2002)
    - Origin of spin alignment? Origin of spin of galaxies?
    - Origin of scatter in star-to-halo ratio?
    - Origin of morphology diversity?
  - Extract relevant information about cosmic web (CW)

→

0^{th} order: mass
1^{st} order: local density
2^{nd} order: "assembly bias" cosmic web anisotropies?



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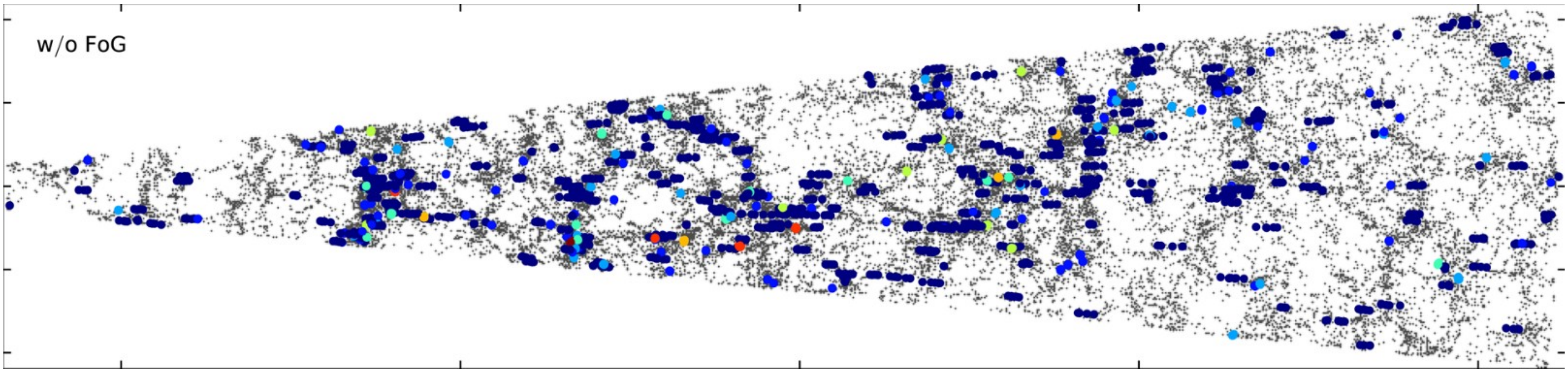

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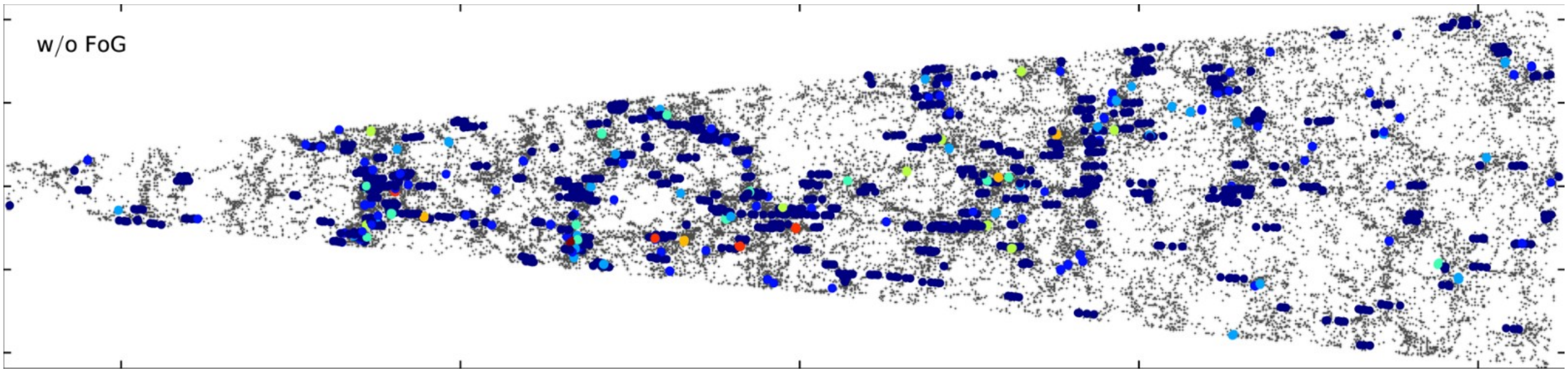
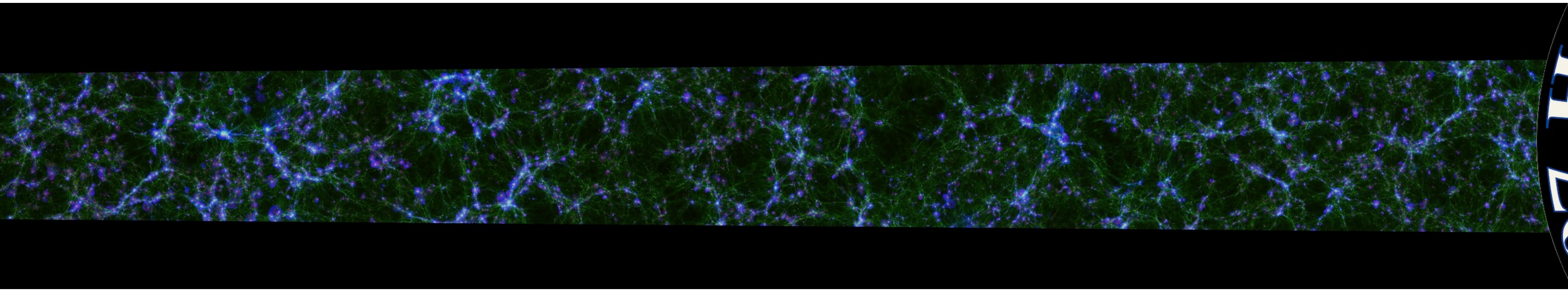


⟹ **Need a way to encode evolution of anisotropy leading to structure formation...**

Motivation(s)

- Unique in terms of (numerical) experiment ⟶ exascale, *i.e.* billions of datapoints to generate
  - How to not be trampled by amount of data?
  - How to compare to observations?
  - What matters and what does not?

→ Use *physically motivated* summary statistics



Slice through the Horizon simulation – not yet exascale, but working on it!

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  - How to not be trampled by amount of data?
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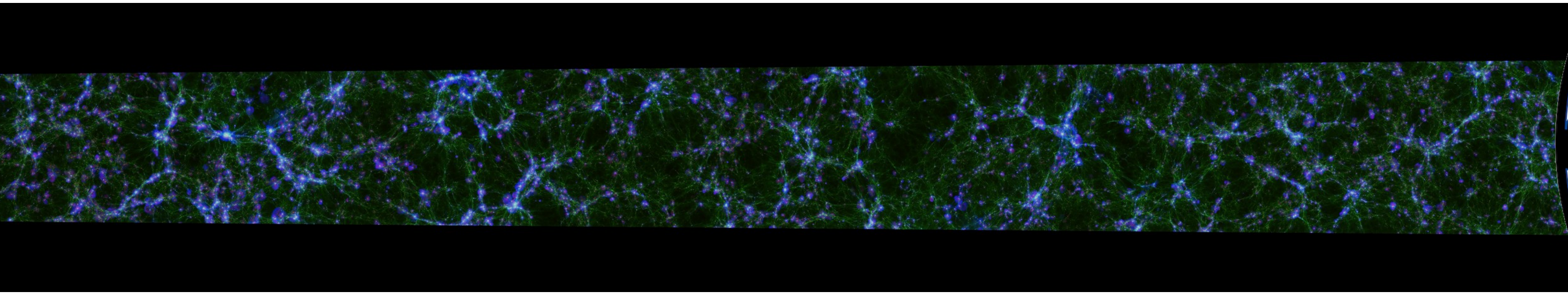
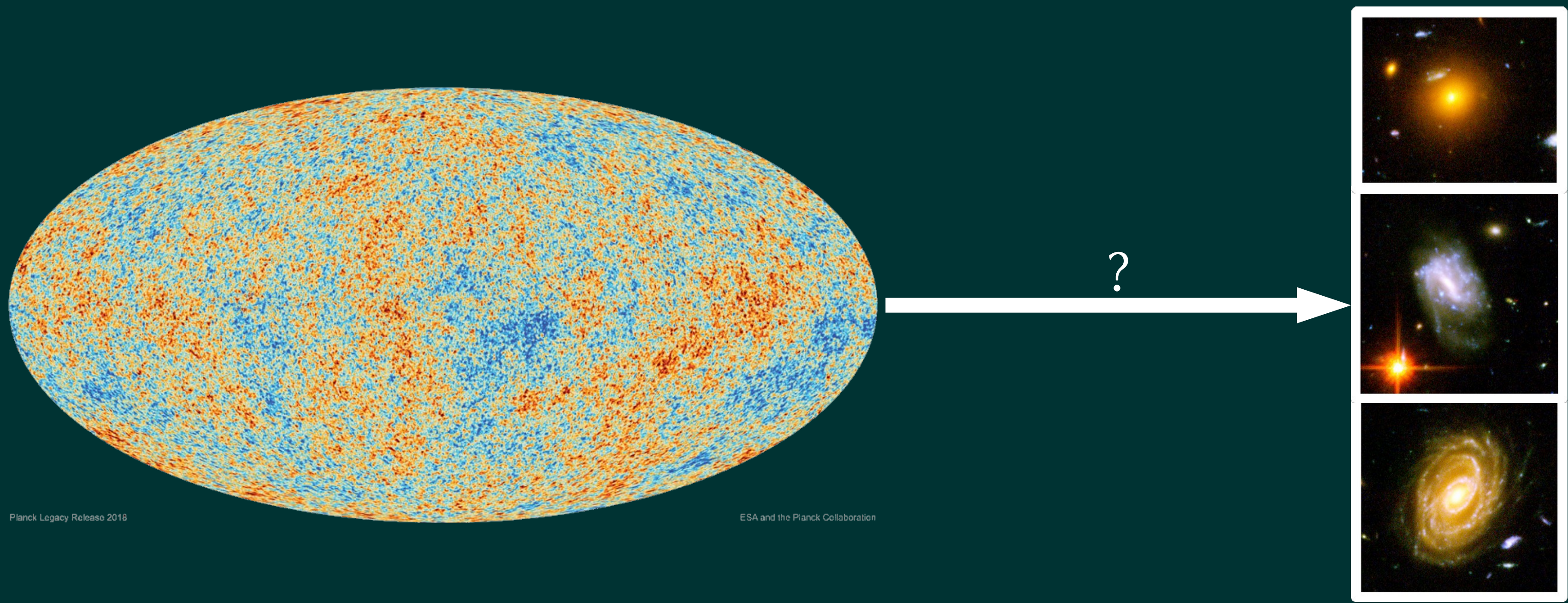
→ Use *physically motivated* summary statistics

⇒ Need a way to encode evolution of anisotropy leading to structure formation…
… in a compact way



Slice through the Horizon simulation – not yet exascale, but working on it!

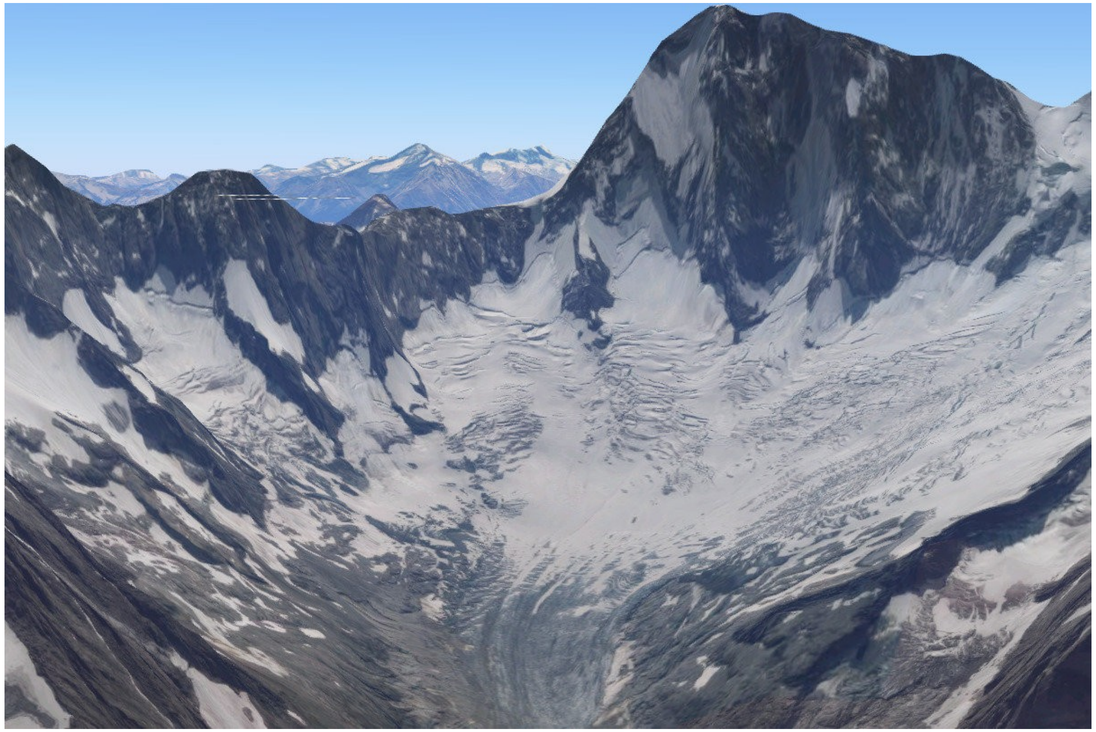
Make predictions in the initial conditions?

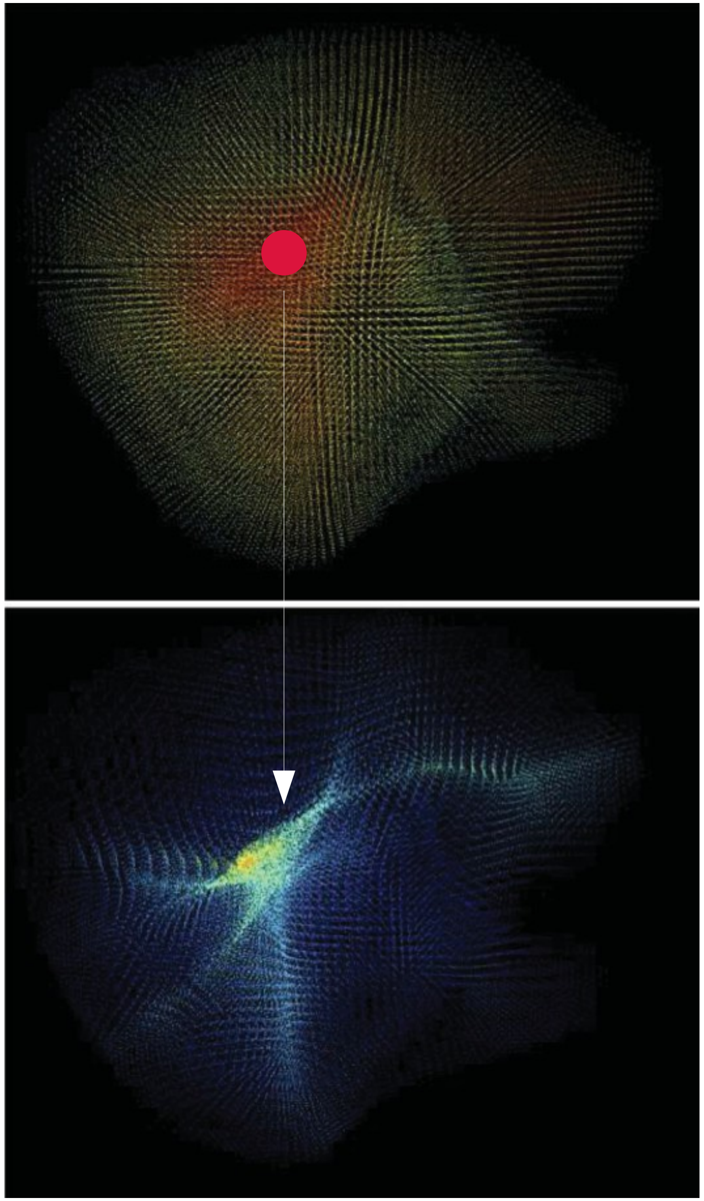



Critical event theory
New Horizon collaboration

Inheriting from BBKS – critical *point* theory

- Proto-halos ~ maxima
- Proto-filaments ~ filament saddle points
- Proto-walls ~ wall-saddle point
- Proto-voids ~ minima





Early time

Late time

Dark matter density in numerical simulation.

[Peak-patch picture: BBKS+86]
[Skeleton theory: Pogosyan+09, ...]

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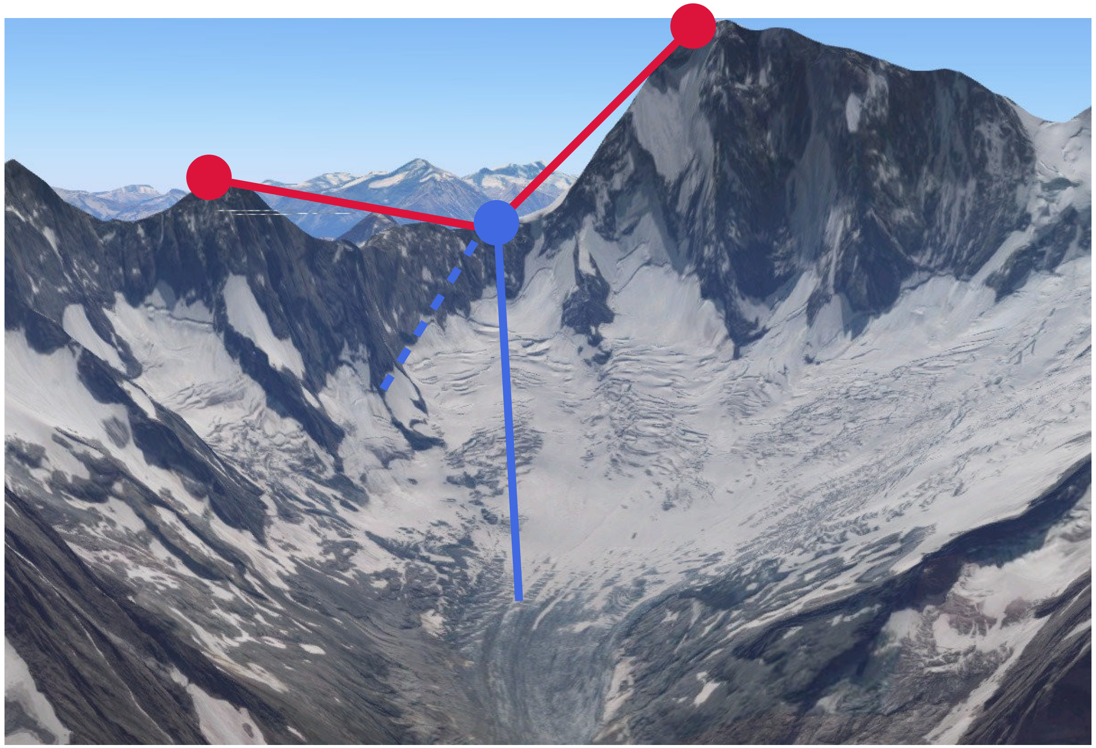

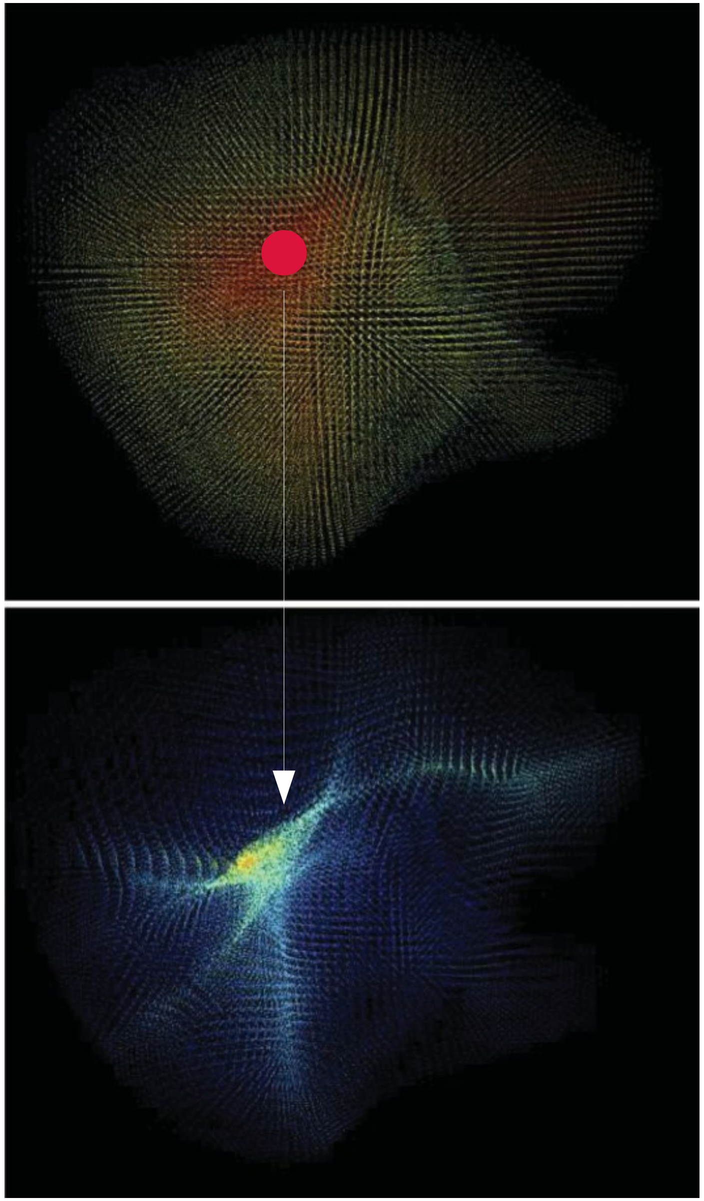

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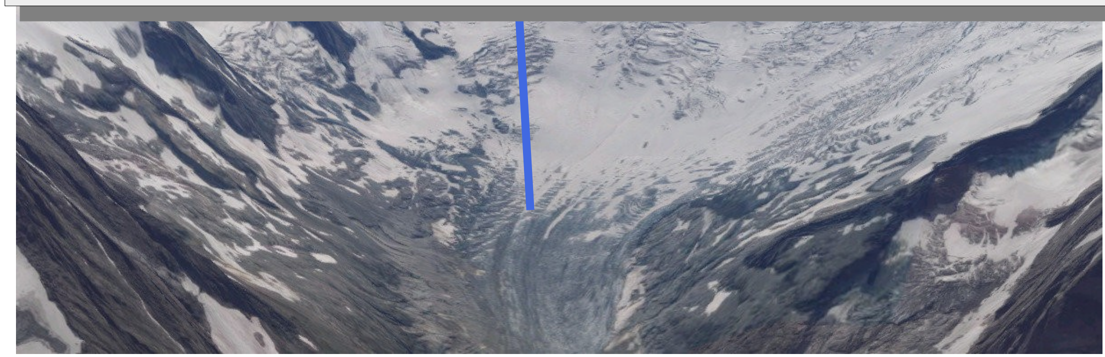
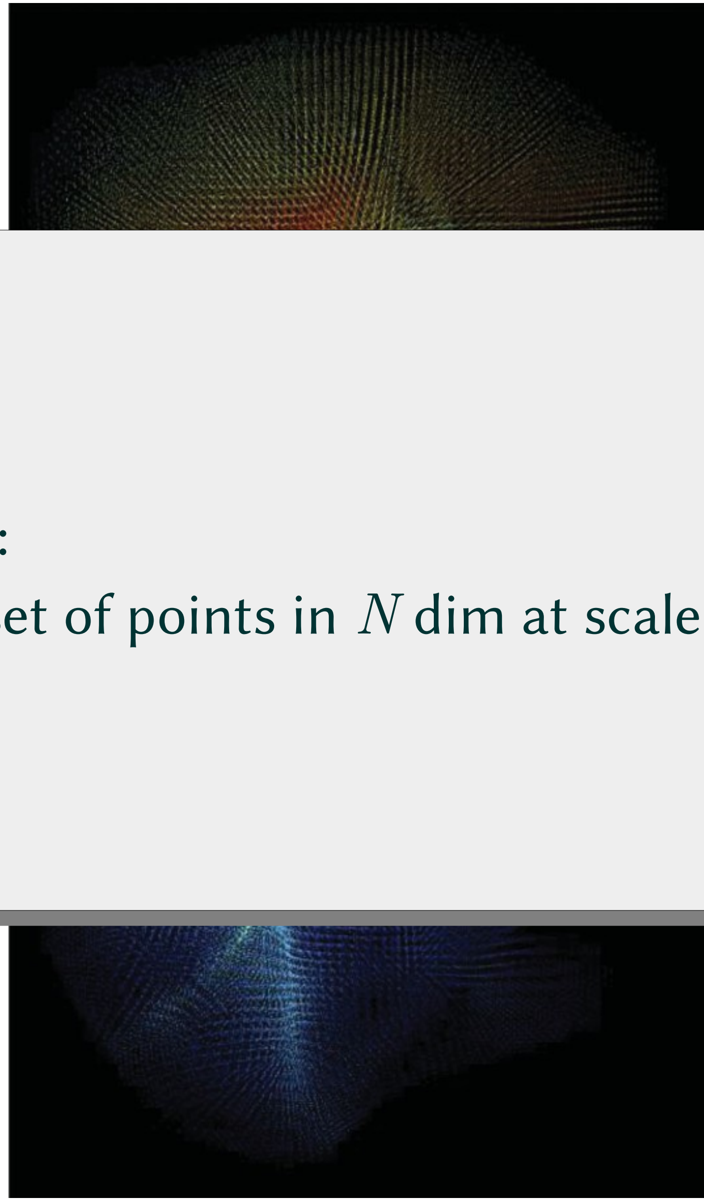
Critical points:

N-dimensional field $\longrightarrow$ compressed in finite set of points in N dim at scale R





Early

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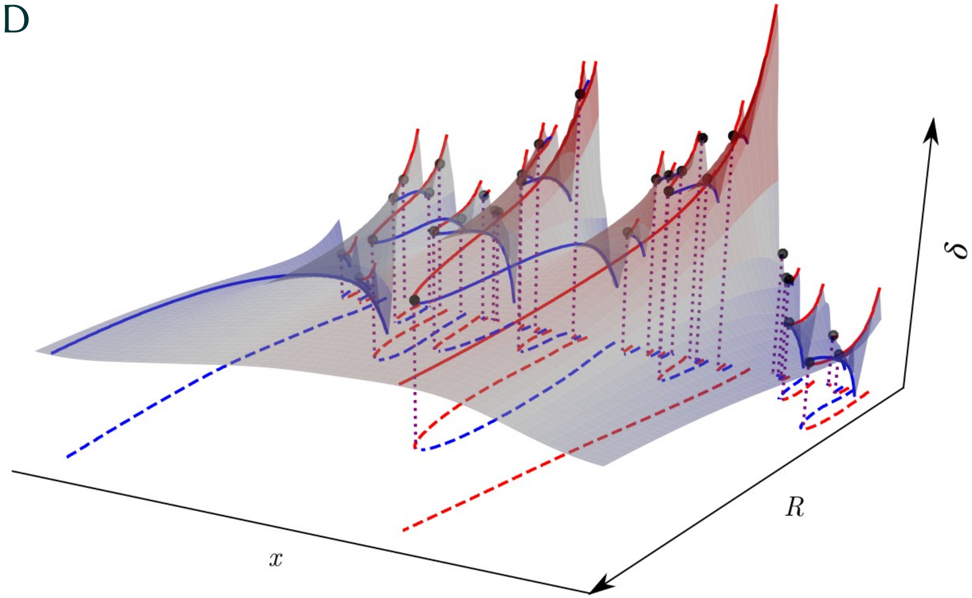
Inheriting from BBKS – critical *event* theory

1+1D



BBKS (peak theory):

Halos form out of peaks

High δ $\longrightarrow$ early formation

High R $\longrightarrow$ high mass

$\Longrightarrow$ multi-scale analysis (different R)

$\longrightarrow$ mass as a function of time

✗ Answer depends on scale considered

✗ Continuous information (*i.e.* $M(z)$)

[Skeleton tree: Hanami+01]

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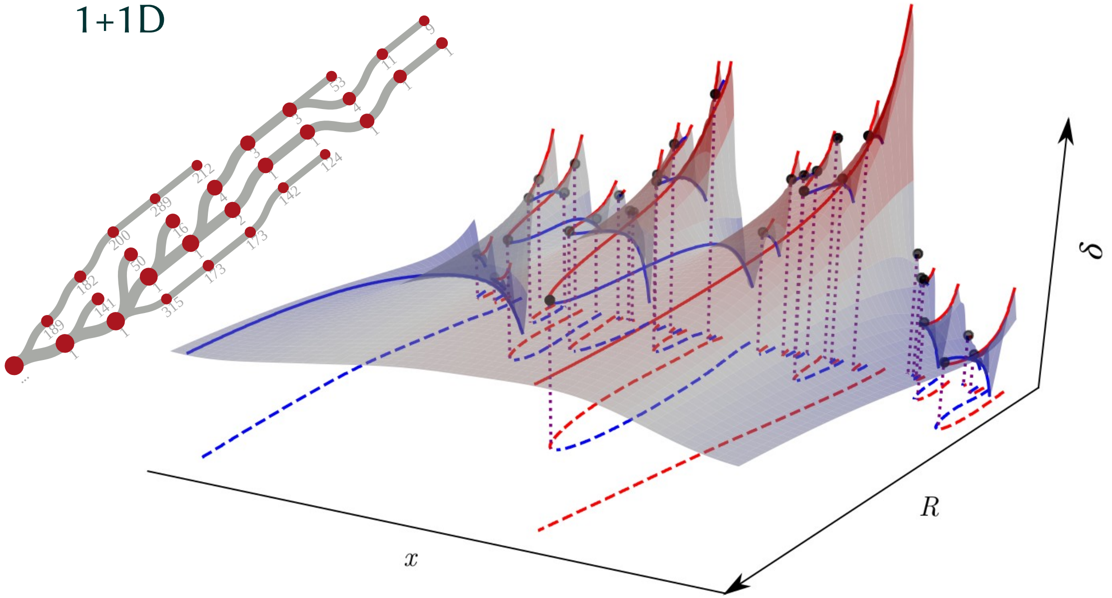

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Critical event theory:

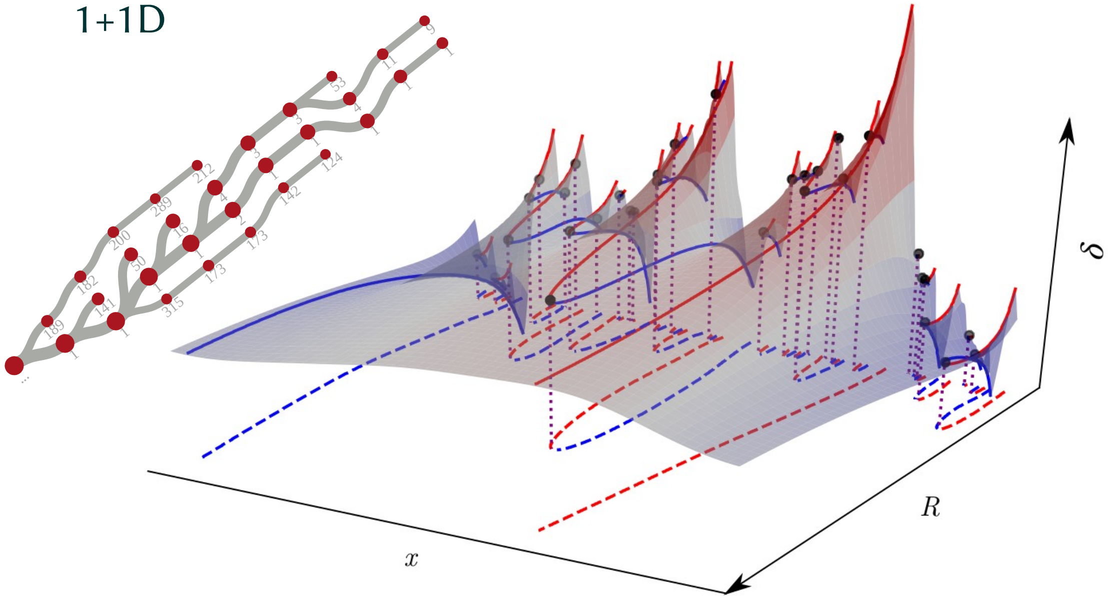
$$\text{``}\,\frac{\partial \mathrm{BBKS}(M(R), t)}{\partial R}\,\text{''}$$

$\Longrightarrow$ Spot peaks disappearing

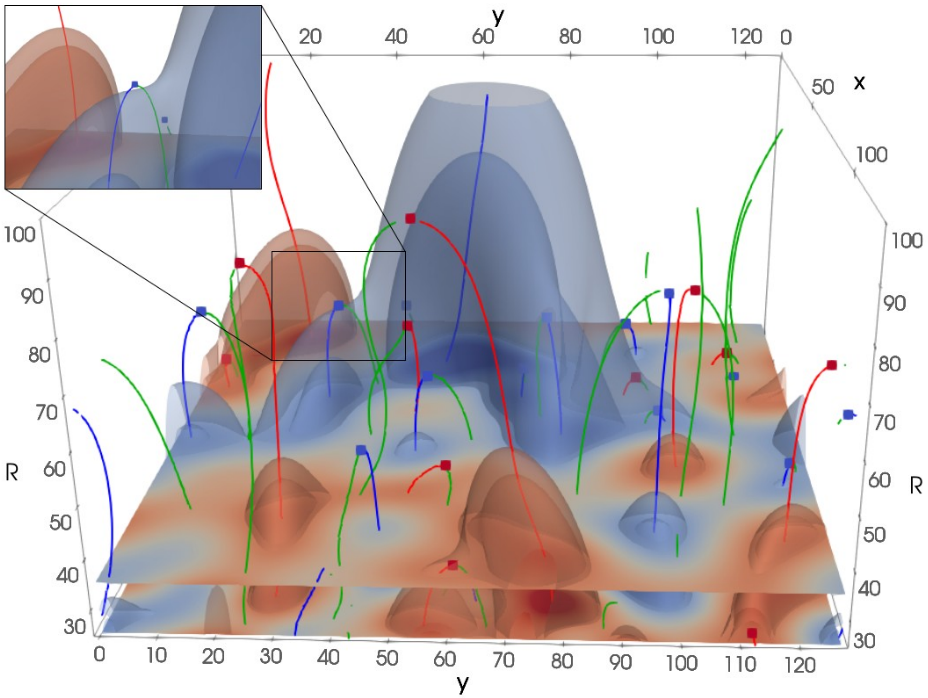
- ✔ Scale intrisic to theory!
- ✔ Efficient compression (*i.e.* $(M(z),z)$)
- ✔ Applicable to peaks, filaments, walls, voids

[Skeleton tree: Hanami+01]

1+1D



2+1D



BBKS (peak theory):

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$\Rightarrow$ multi-scale analysis (different R)

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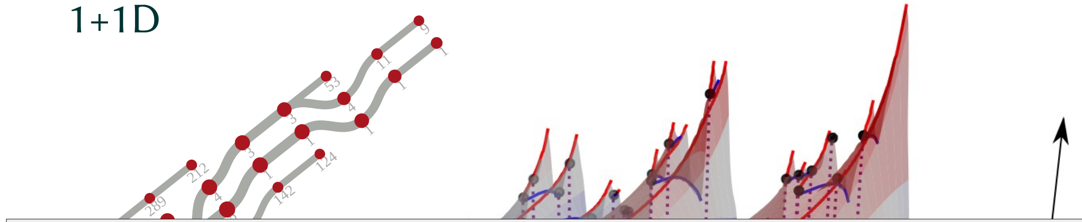

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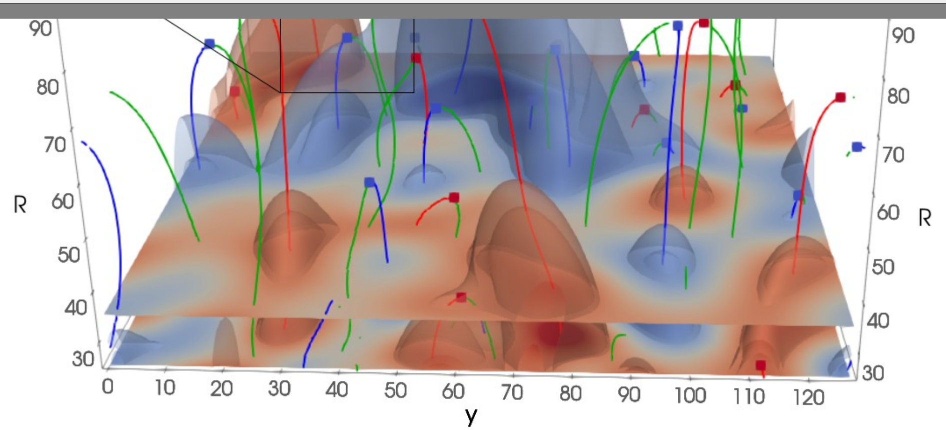
N-dimensional field ⟶ compressed in finite set of points in N+1 dim



∂R

⟹ Spot peaks disappearing

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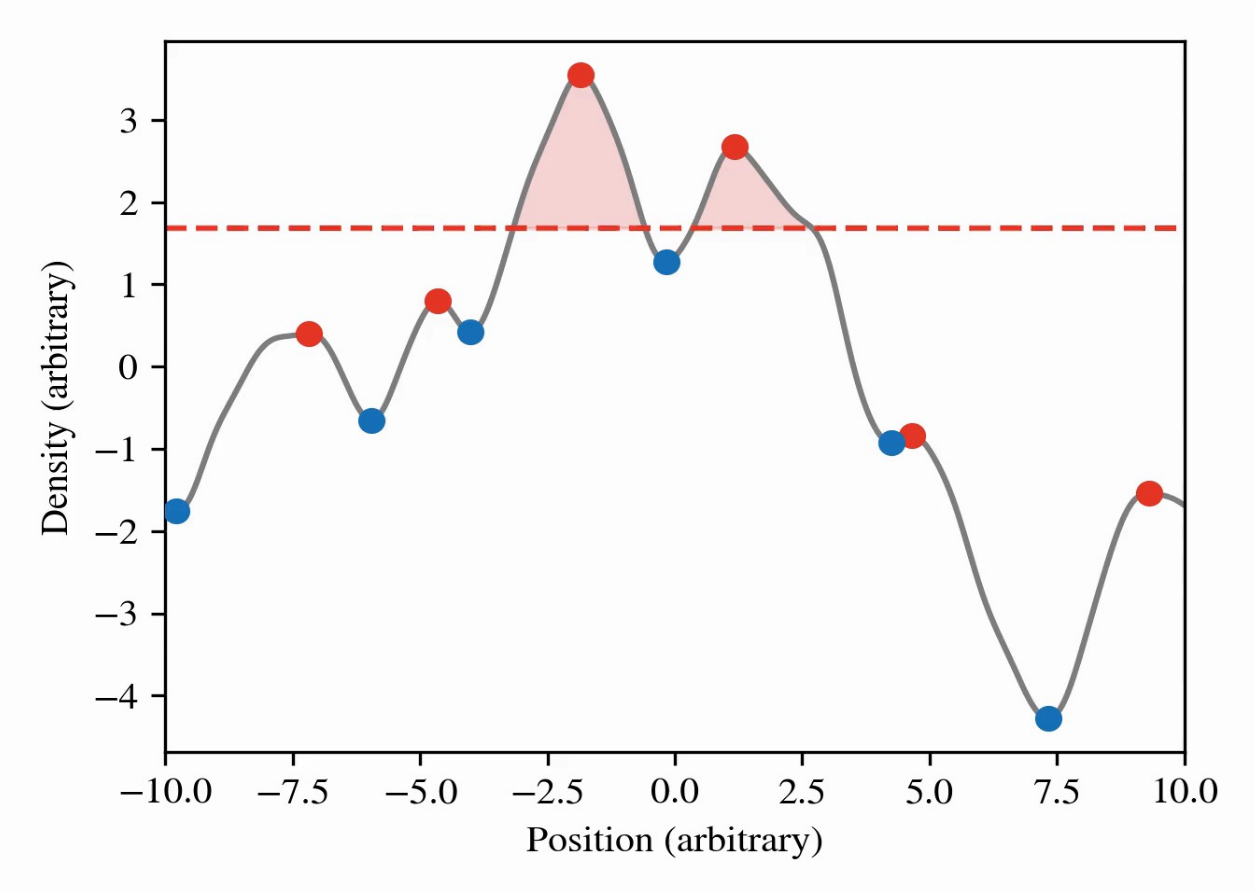
[Skeleton tree: Hanami+01]

Critical events – 1+1D case



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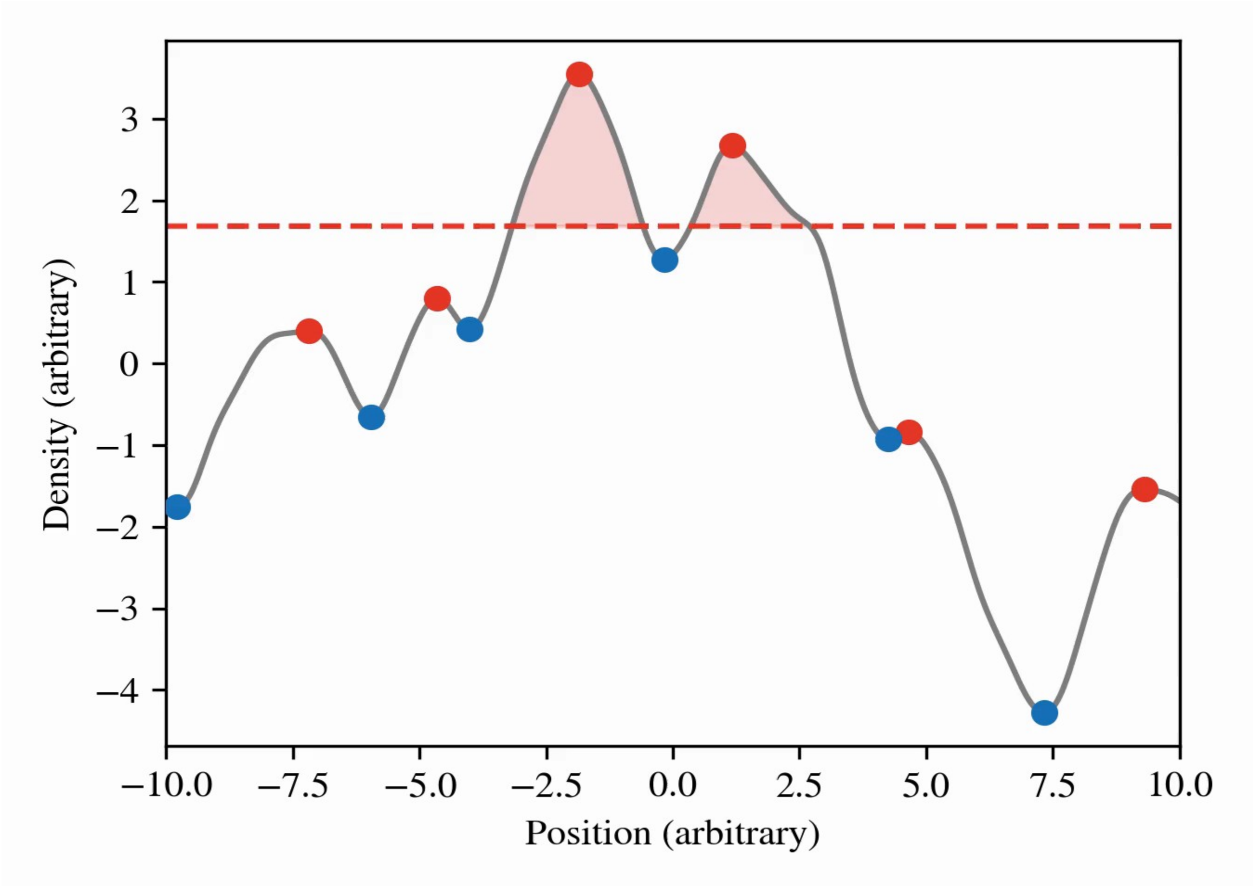

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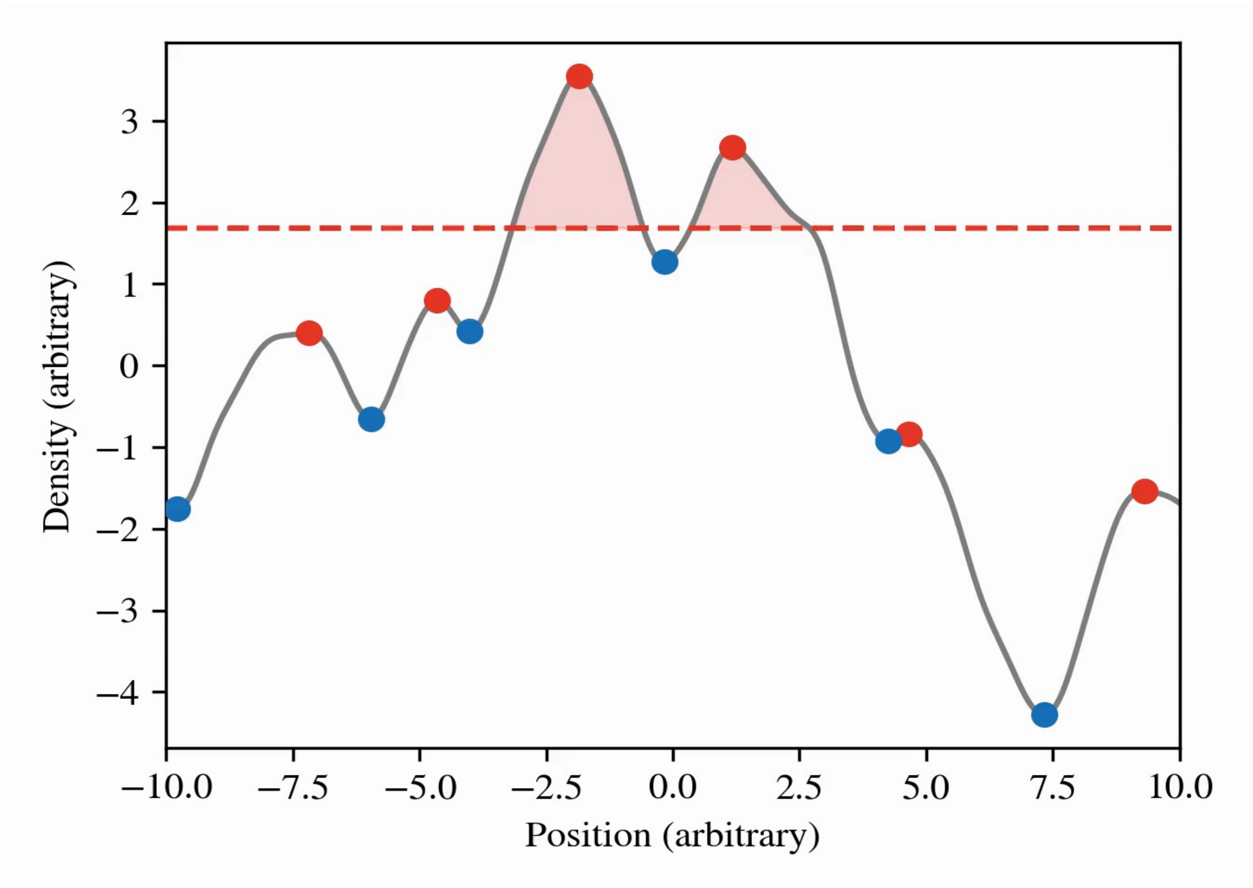
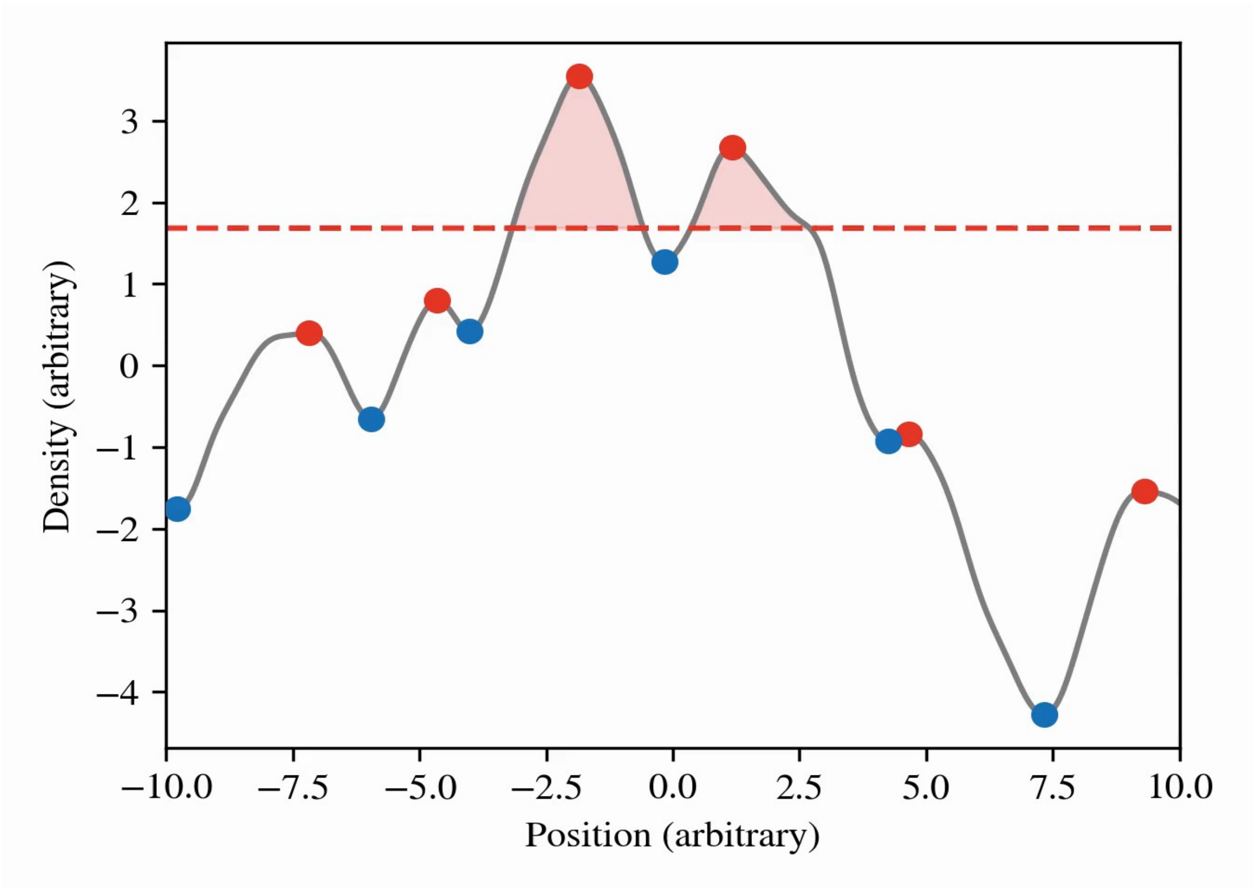


peak +
saddle + → larger peak
peak

In the initial conditions (Lagrangian space)

Critical events – 1+1D case



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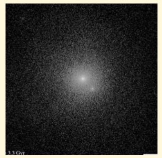
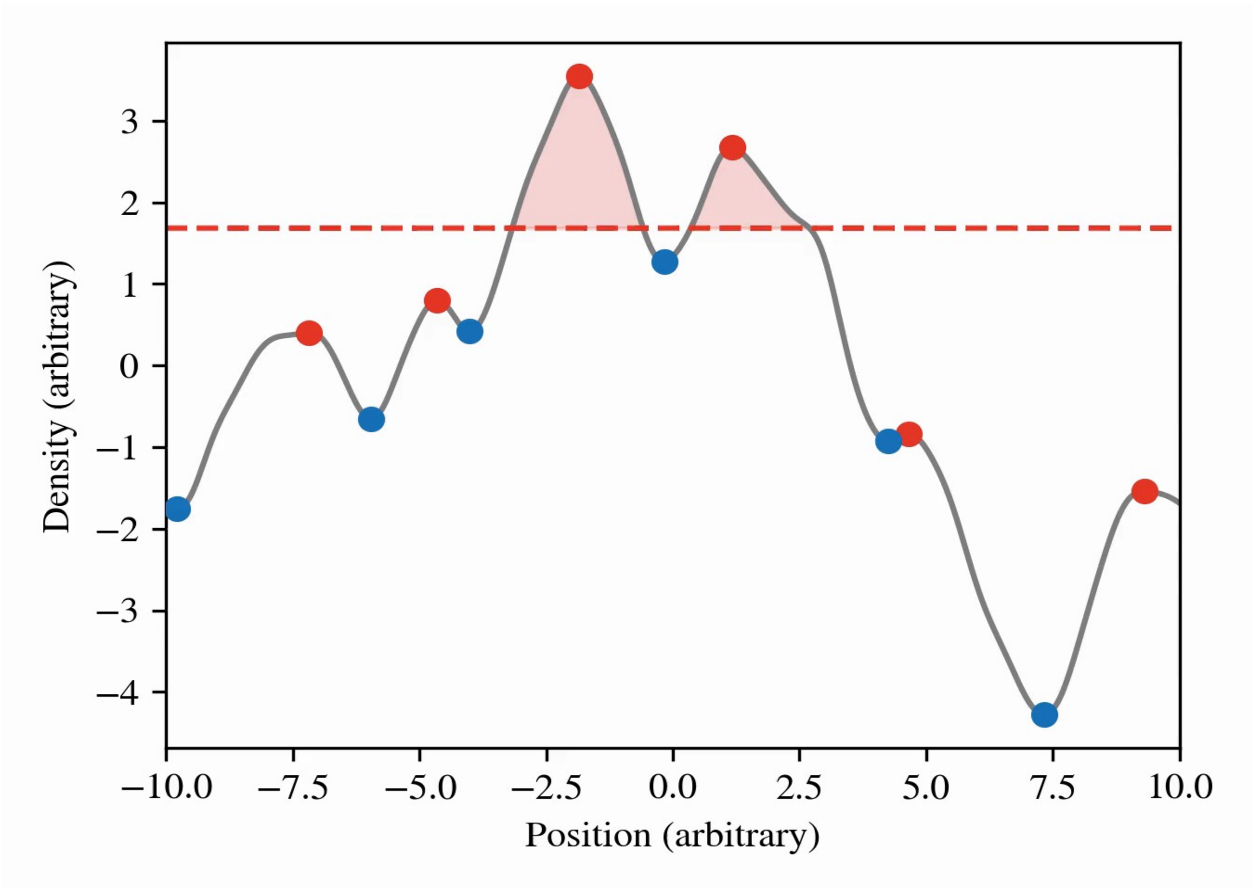
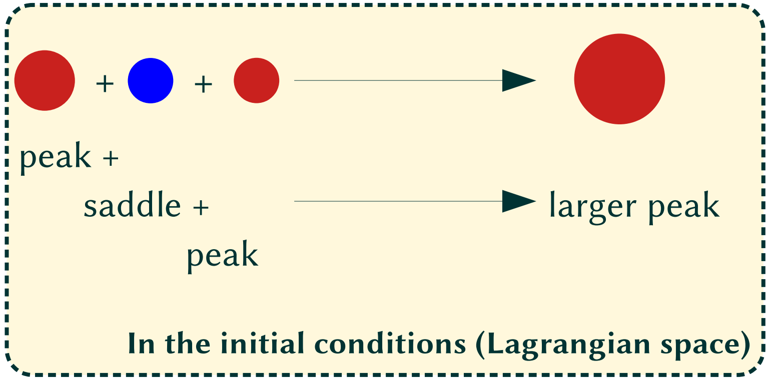
  Halo merger → 

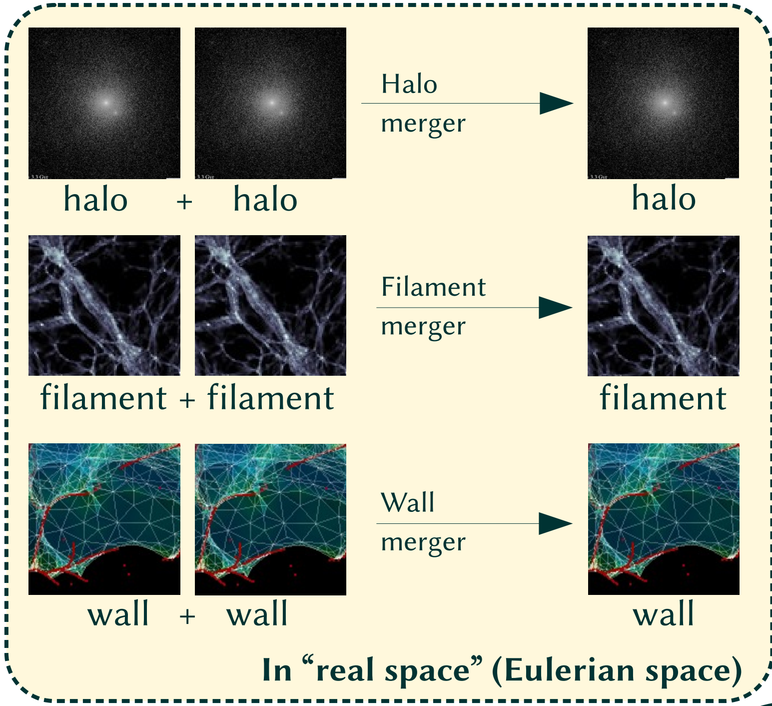
halo + halo → halo

In "real space" (Eulerian space)

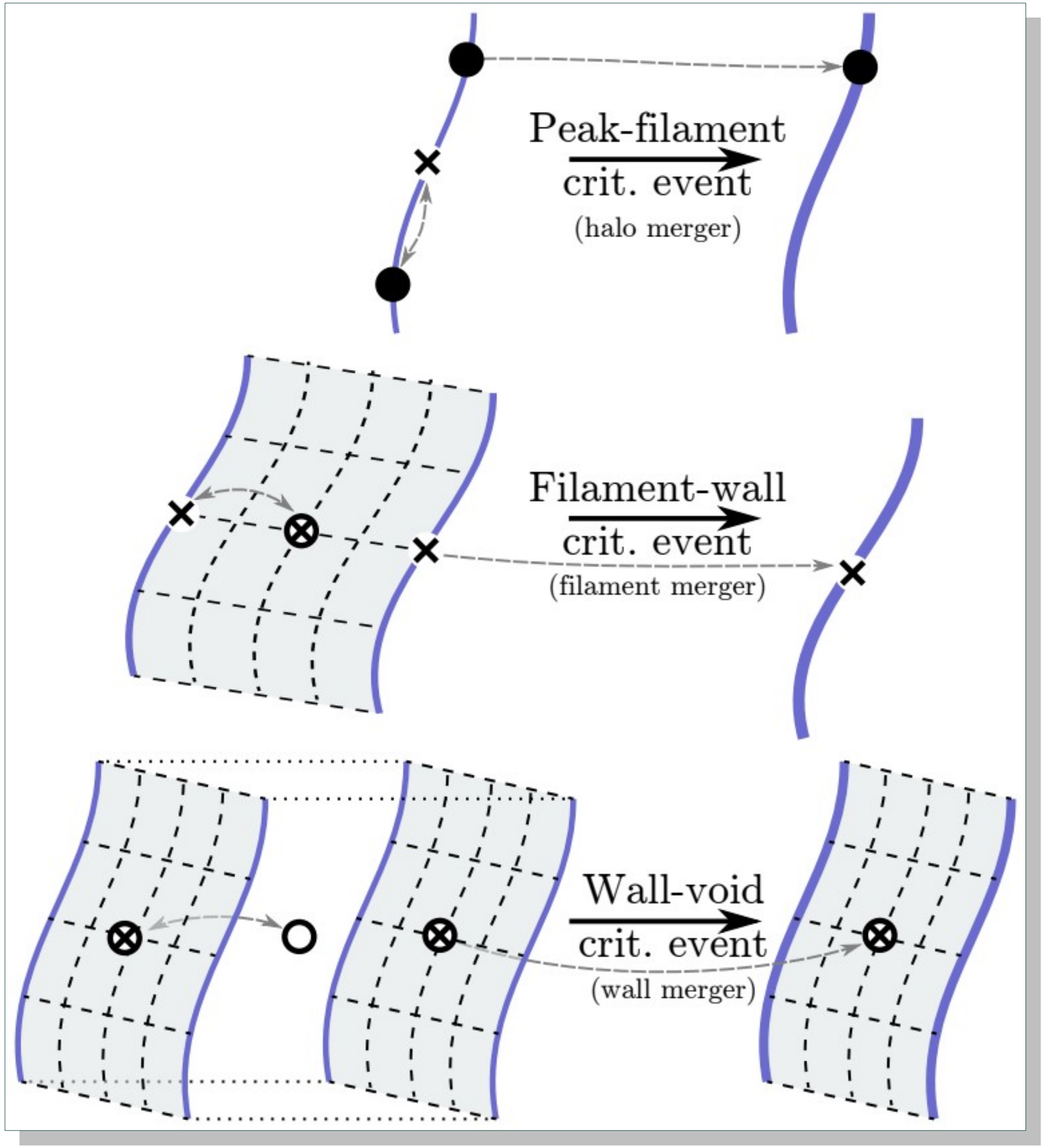
Critical events – 1+1D case





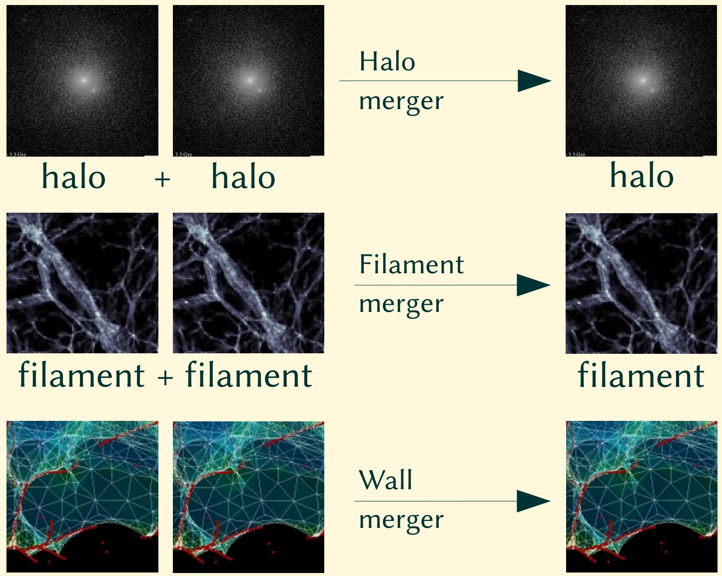


Critical events – 3+1D case



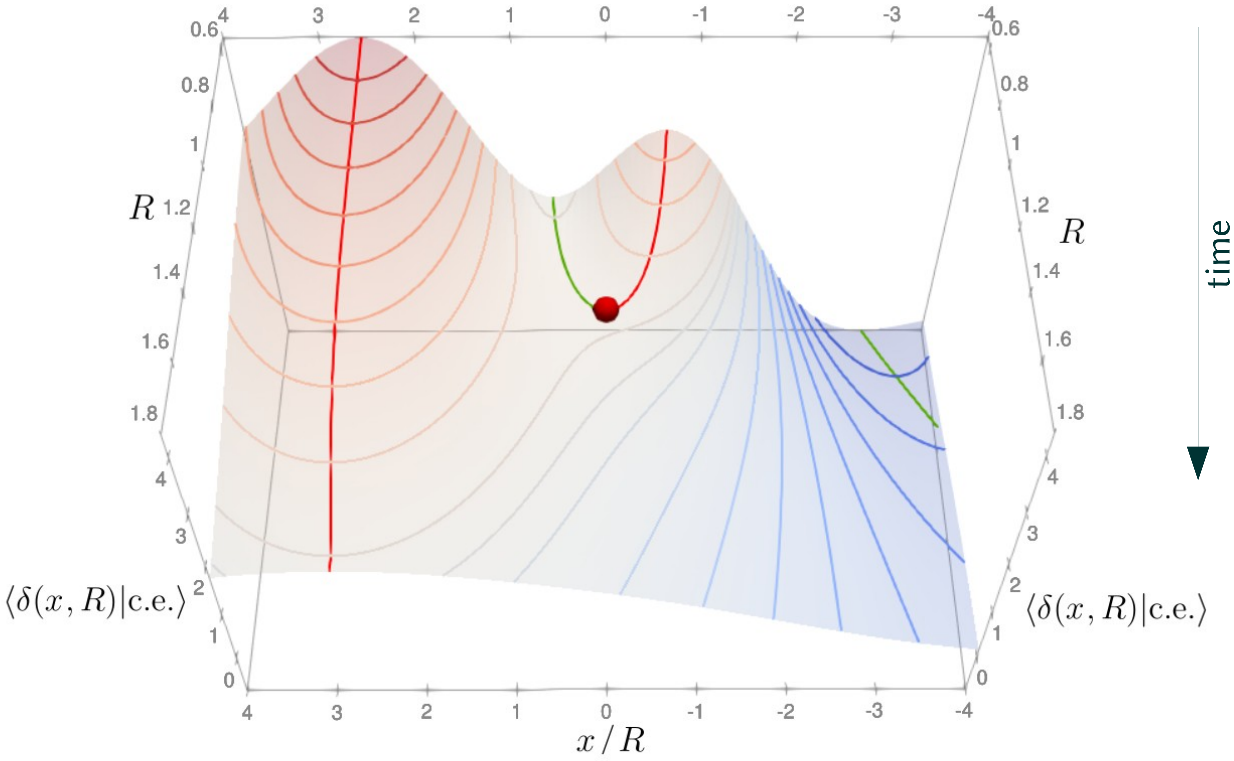
peak +
saddle +
peak
larger peak

In the initial conditions (Lagrangian space)

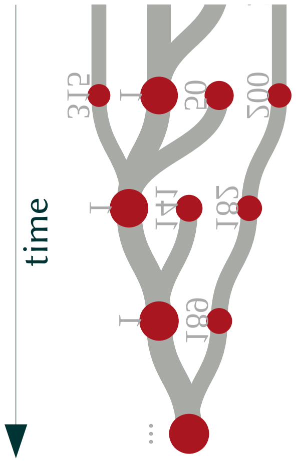


In "real space" (Eulerian space)

Critical events – two possibilities



Mean density field near a critical event



Destruction critical events
⟶ predictors for mergers

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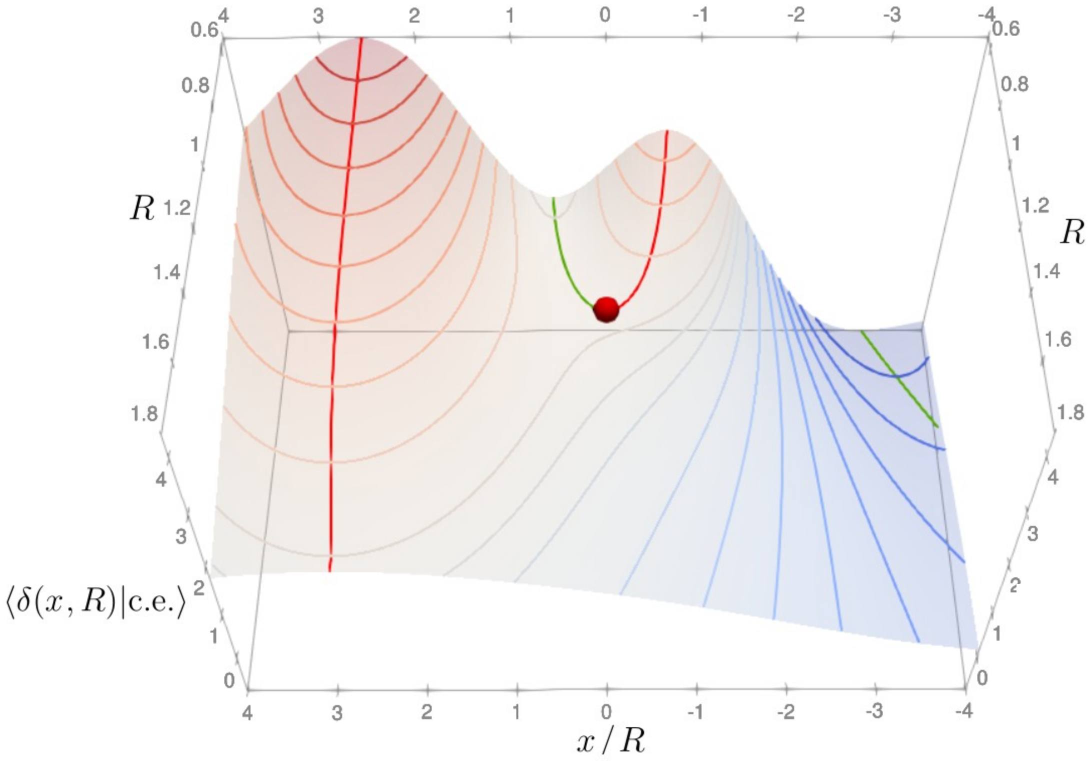

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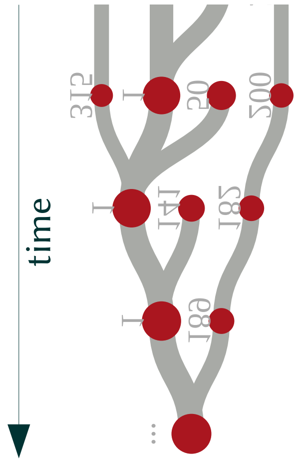

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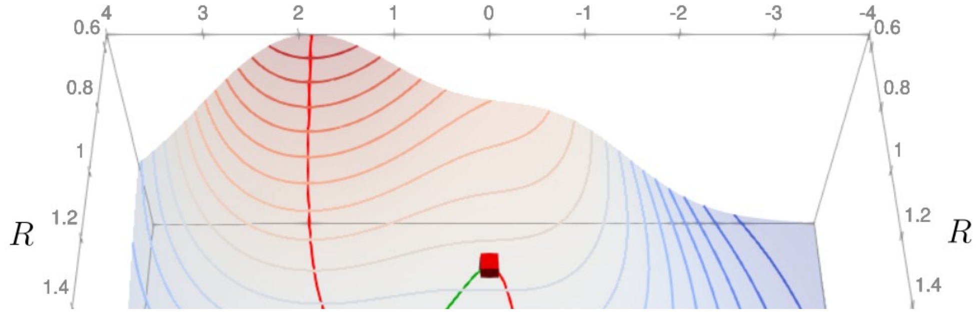
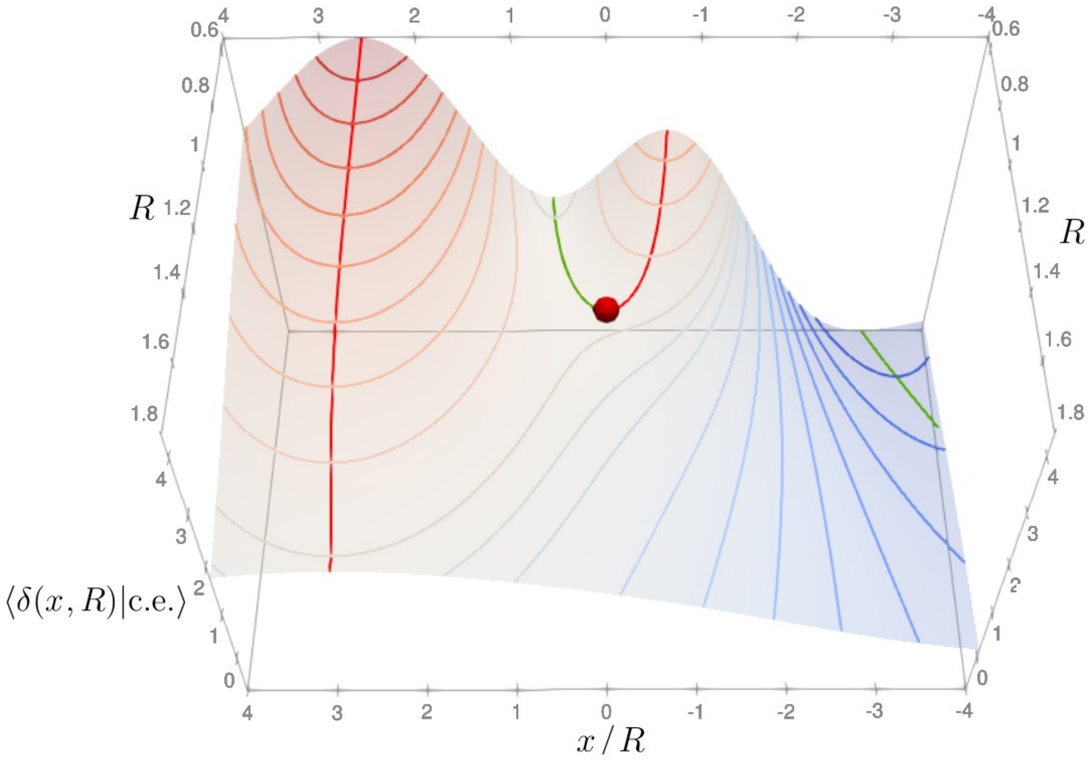


Nucleation critical events
⟶ splashback halos?
⟶ transients?
(50x less frequent, only possible in ≥2D)

interactive viz

Critical events – two possibilities



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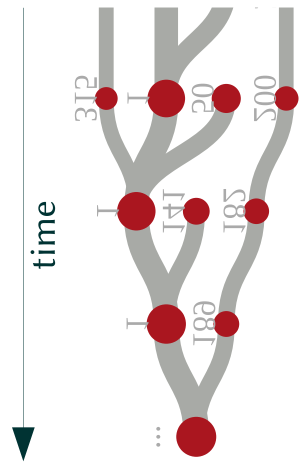

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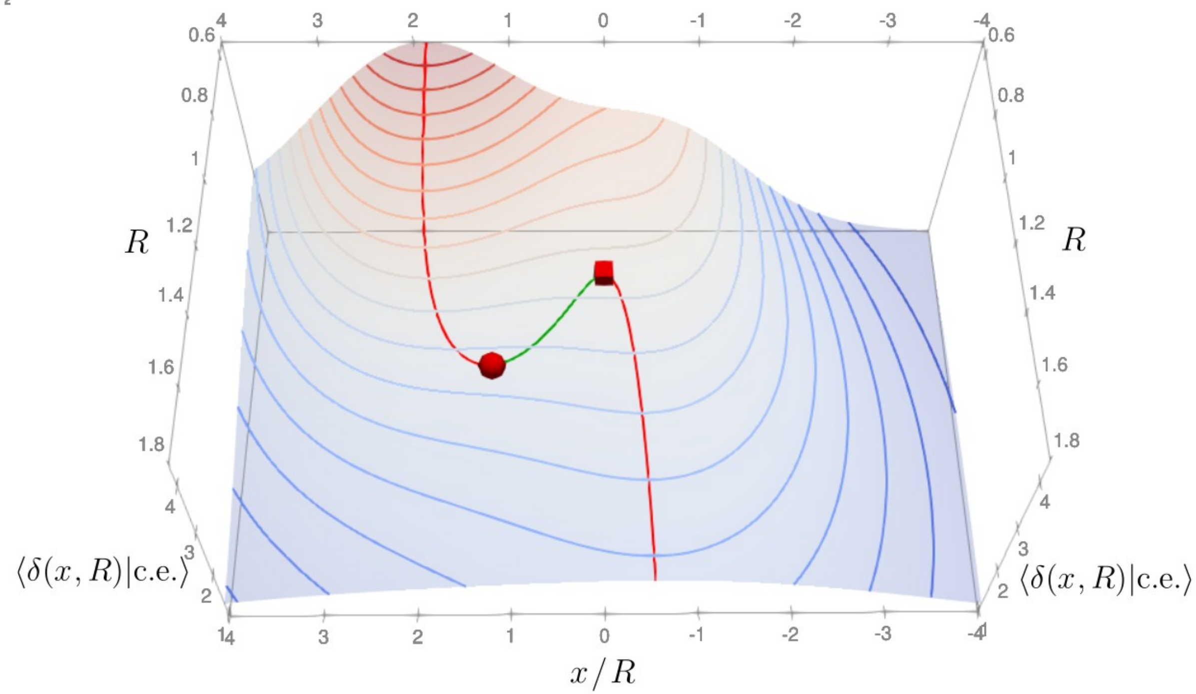

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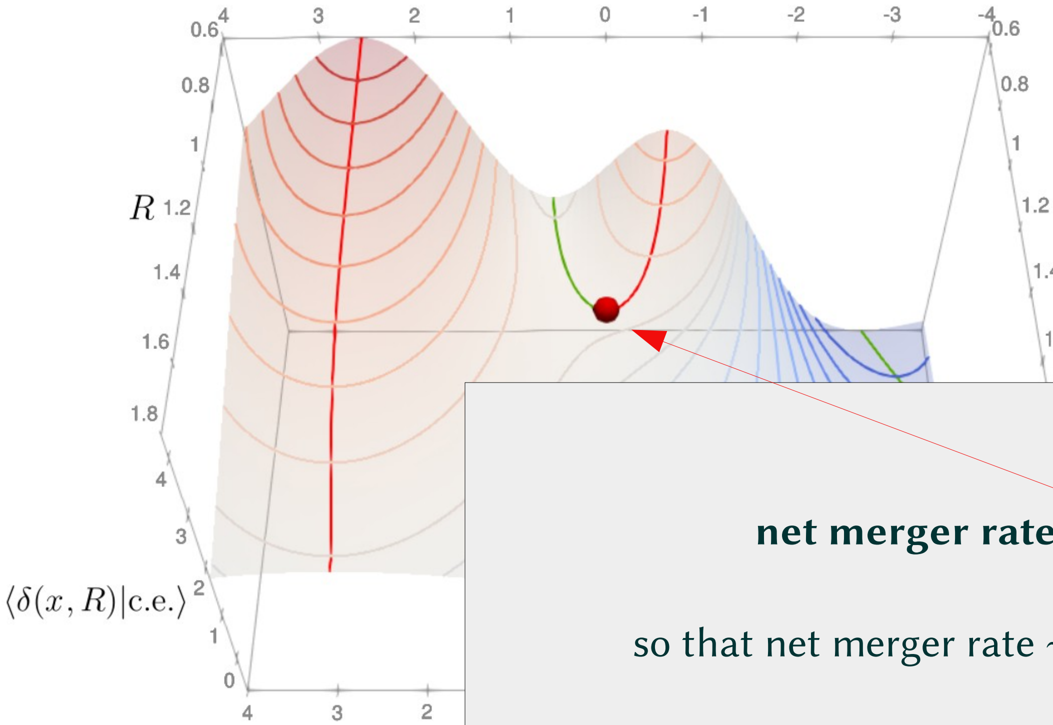

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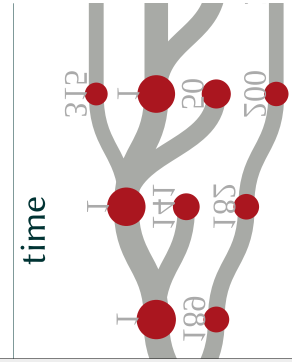

Destruction critical events
⟶ predictors for mergers

net merger rate = - destruction + nucleation

so that net merger rate ~ derivative of number of critical point

Nucleation critical events
⟶ splashback halos?
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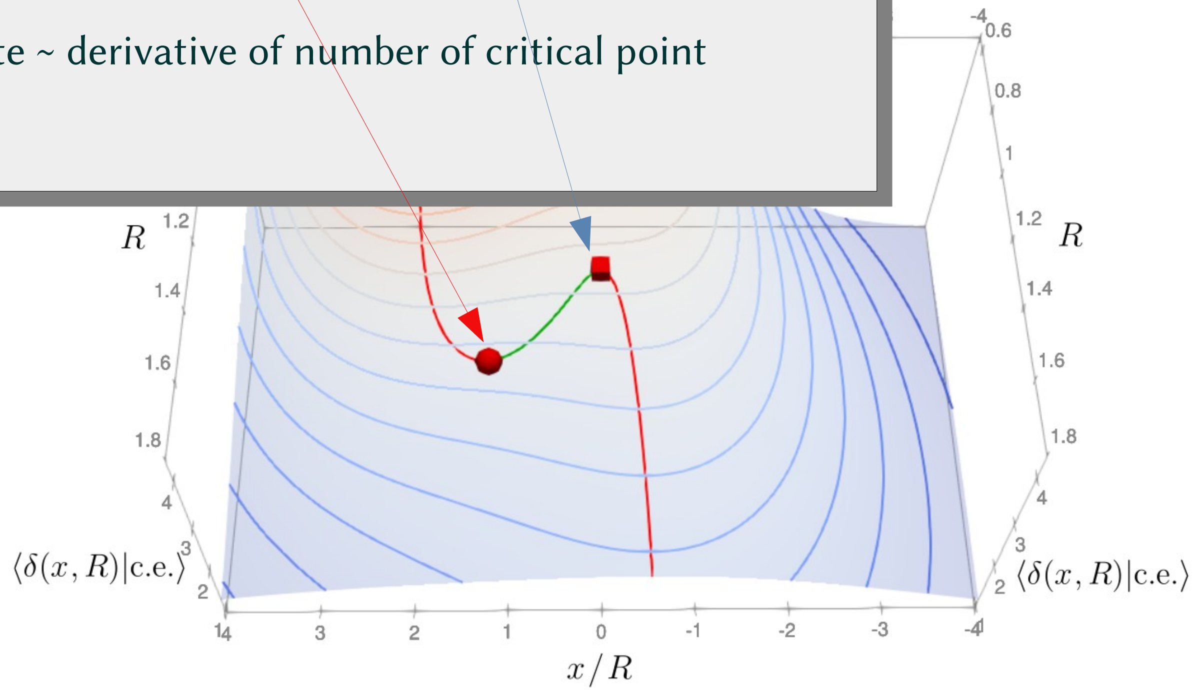


interactive viz

Net merger rates in 3D

$$n_{\rm me}^{(j)} = \frac{R}{\tilde{R}^2 R_*^3} C_{\rm odd}\, C_{j,\rm even}\,,$$

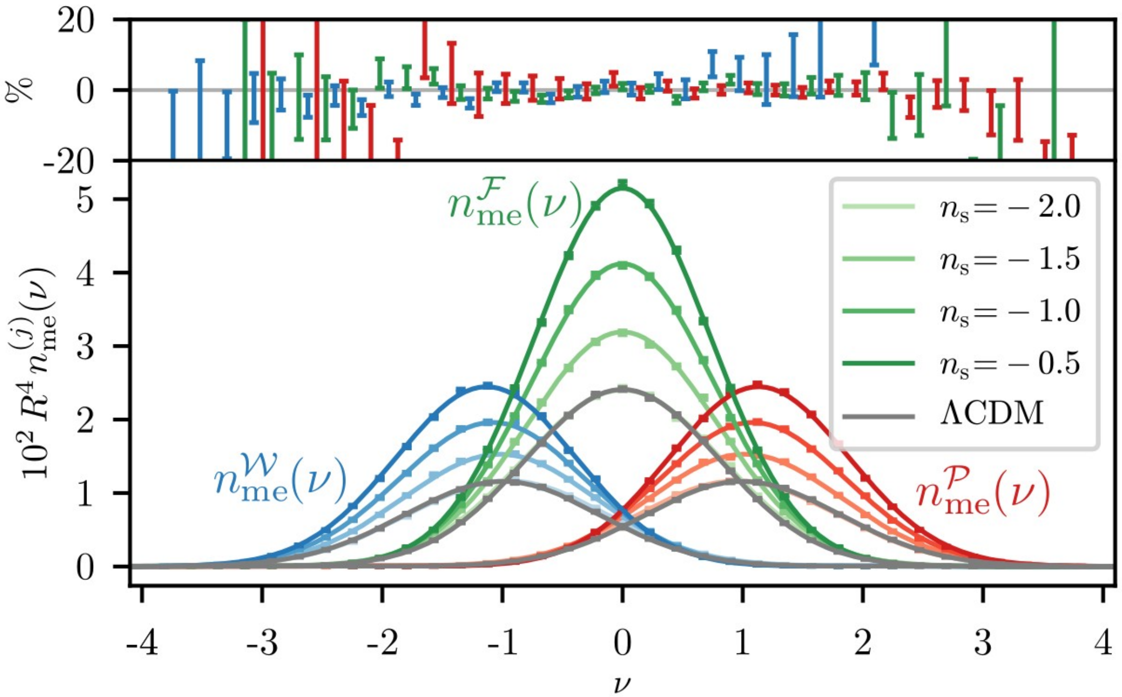
$$C_{3,\rm even}(\nu) = \sum_{i=5,6,9} c_{3,i} \exp\left(-\frac{\nu^2}{2(1-5\gamma^2/i)}\right),$$
$$C_{2,\rm even}(\nu) = c_{2,6} \exp\left(-\frac{\nu^2}{2(1-5\gamma^2/6)}\right),$$
$$C_{1,\rm even}(\nu) = C_{3,\rm even}(-\nu), \tag{39}$$

with

$$c_{3,5} = \frac{3\sqrt{5}\gamma\nu\sqrt{1-\gamma^2}\left(275\gamma^4 + 30\gamma^2\left(2\nu^2-23\right)+351\right)}{\pi\sqrt{2\pi}\left(9-5\gamma^2\right)^4},$$

$$c_{3,6} = -\frac{\mathrm{erf}\left(\frac{\gamma\nu}{\sqrt{2(1-\gamma^2)(6-5\gamma^2)}}\right)+1}{\sqrt{5}\pi\sqrt{6-5\gamma^2}}, \quad c_{2,6} = \frac{2}{\sqrt{5}\pi\sqrt{6-5\gamma^2}},$$

$$c_{3,9} = \frac{\mathrm{erf}\left(\frac{\sqrt{2}\gamma\nu}{\sqrt{(1-\gamma^2)(9-5\gamma^2)}}\right)+1}{4\pi\sqrt{5}\left(9-5\gamma^2\right)^{5/2}} \times \left(\frac{3600\gamma^4\nu^4}{(9-5\gamma^2)^2} + \frac{120\gamma^2\left(27-35\gamma^2\right)\nu^2}{9-5\gamma^2} + 575\gamma^4 - 1230\gamma^2 + 783\right).$$

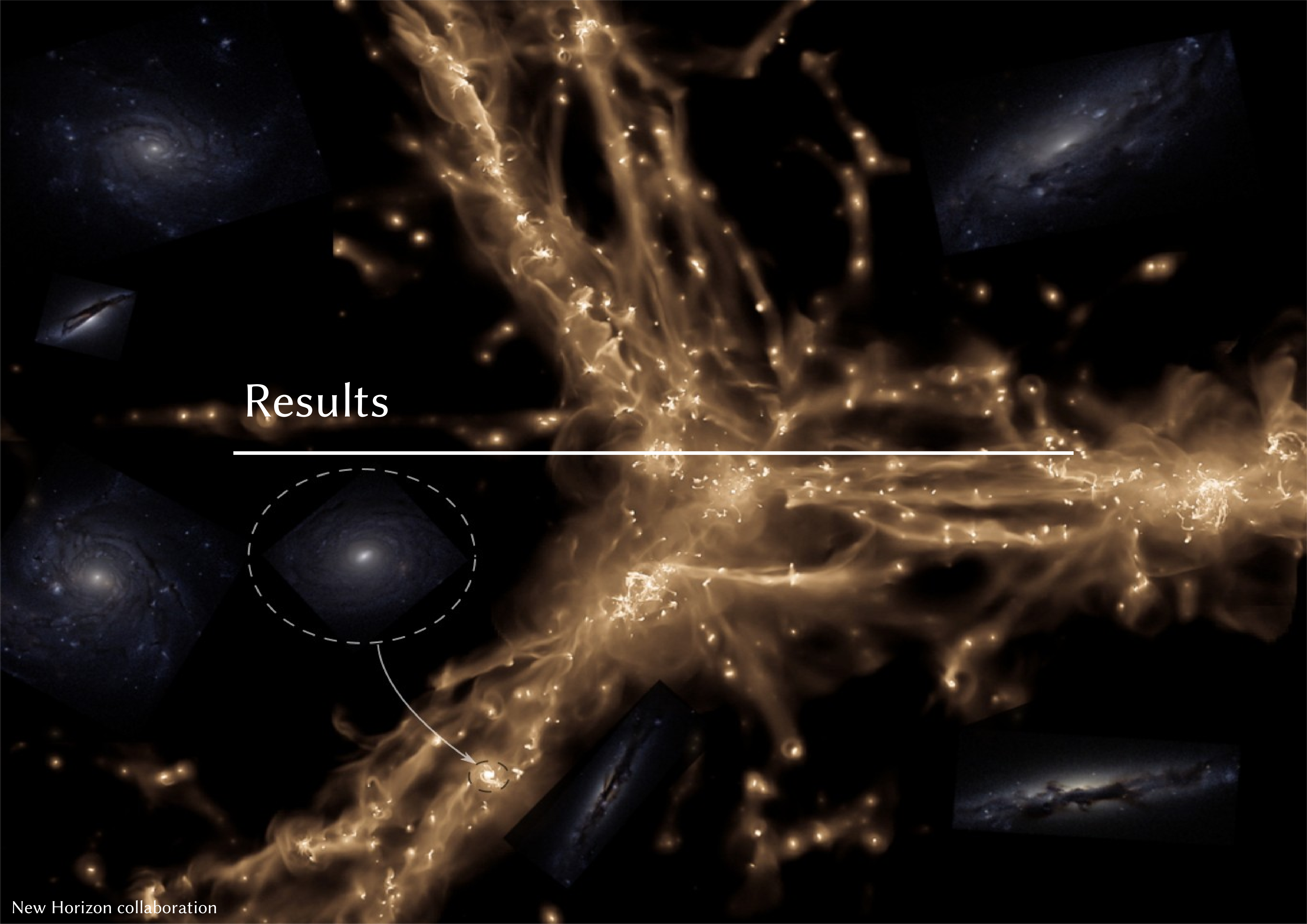


Net merger rate for peaks (P), filaments (F) and walls (W).

Take home messages:

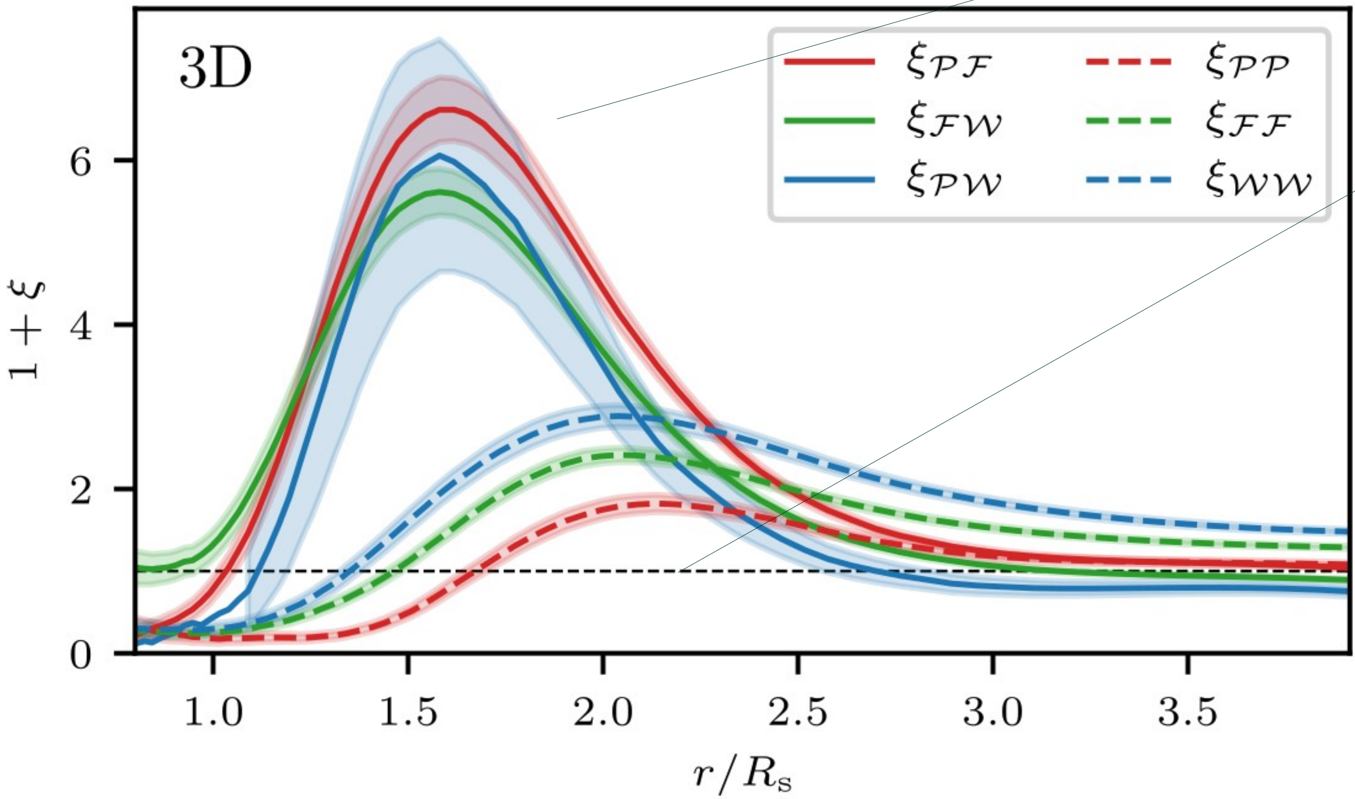
1) We can compute merger rates in the initial conditions…
2) … and measure them in GRF data cubes…
3) … and all of that agree!

Results



New Horizon collaboration

On the problem of connectivity



Clustering of **peak mergers** and **filament mergers**

Exclusion zone of **peak merger** with **peak merger**

Ratio of peak positions ~ 5/4
~ 4 filaments mergers can be packed before next peak merger*

* $2.055 \times (5/4)^3 \cong 4$

On the problem of connectivity

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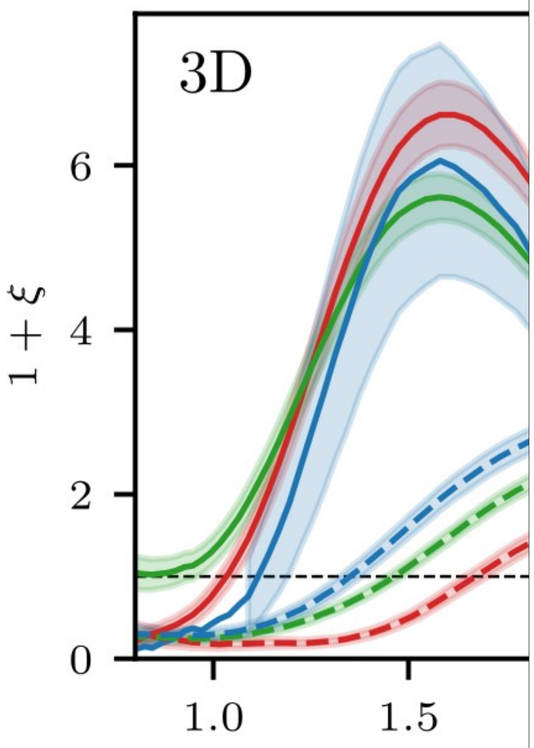
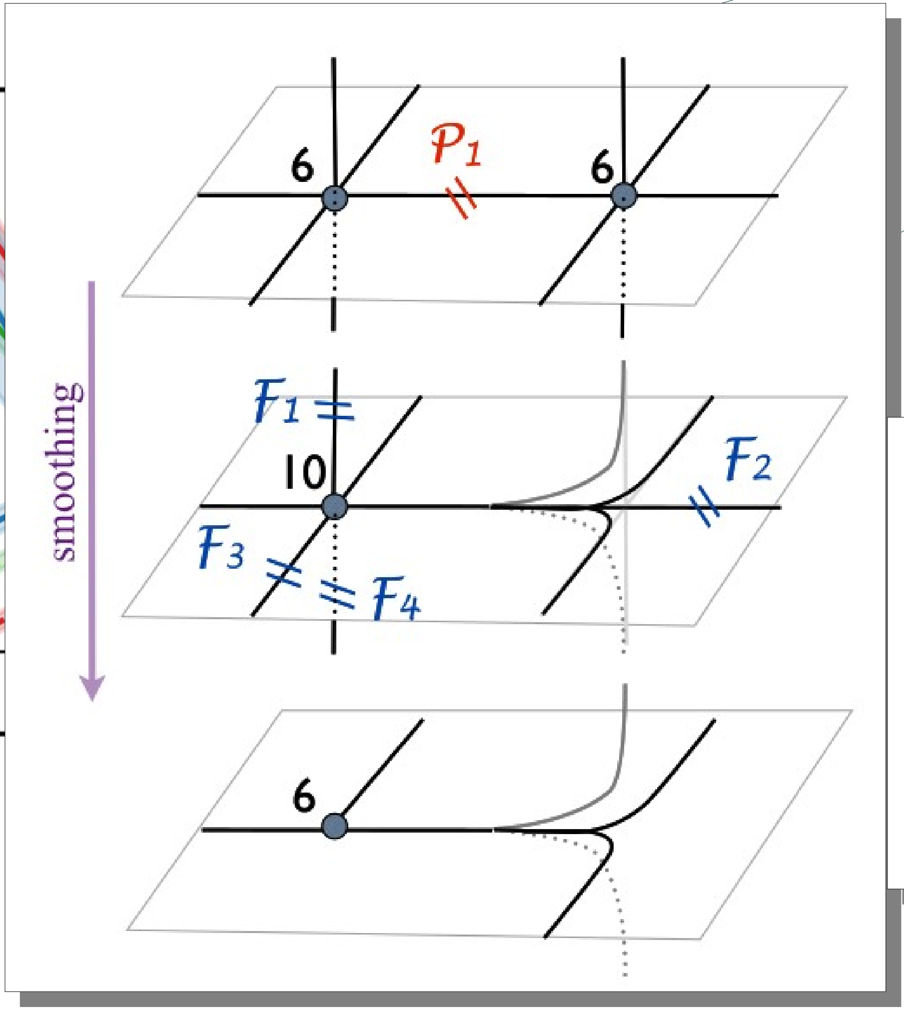




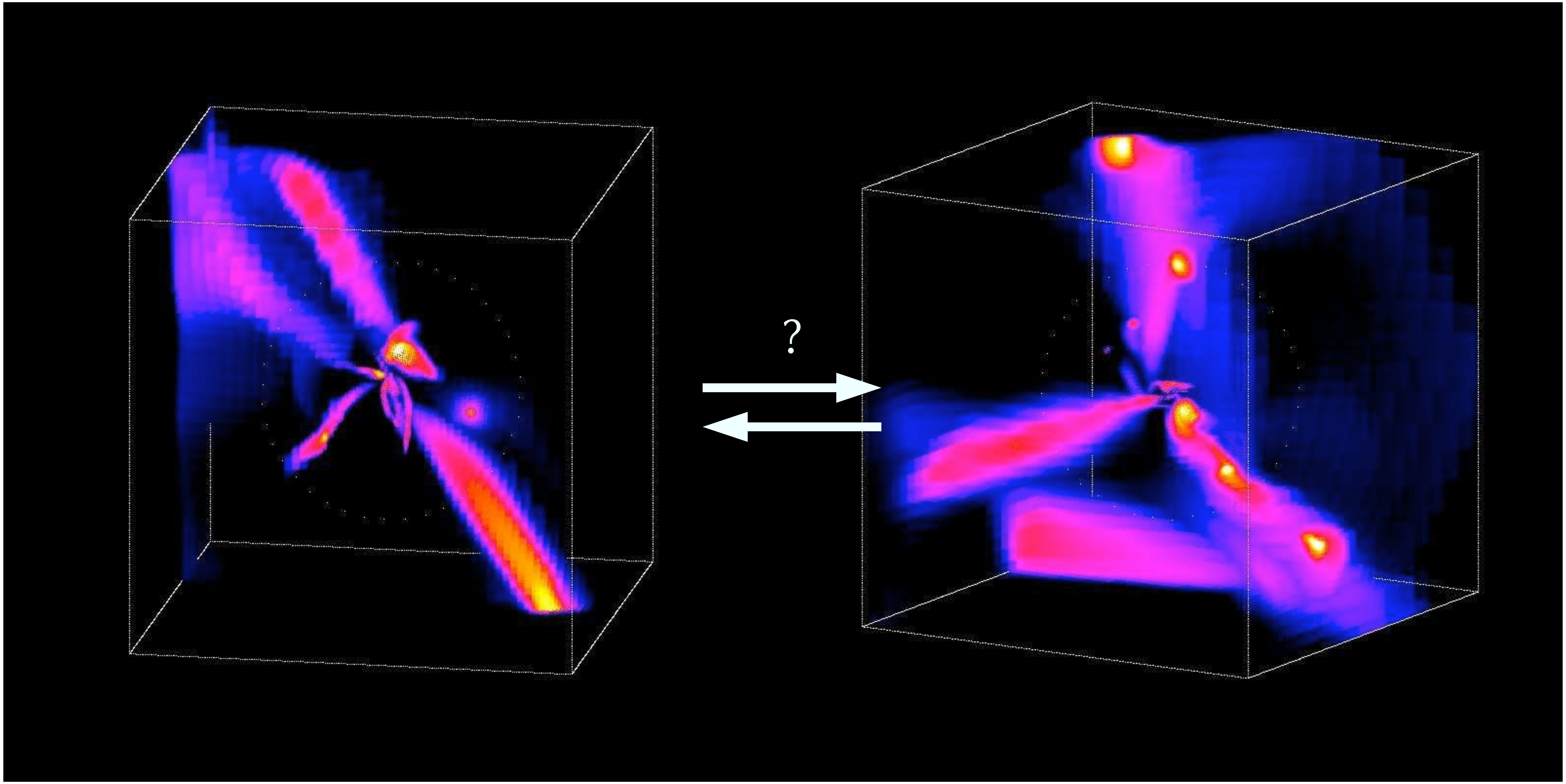
Likely timeline $\mathcal{P}\mathcal{F}^4\mathcal{P}$ sequence:

1) One halo merger
2) Four filament mergers (each disconnects two sides)
3) One halo merger (outside of pic.)

Crit. events ⟶ preserve connectivity

* 2.055 x $(5/4)^3 \cong 4$

On the problem of connectivity



[Dekel+09; Codis+18 for predictions of the connectivity]

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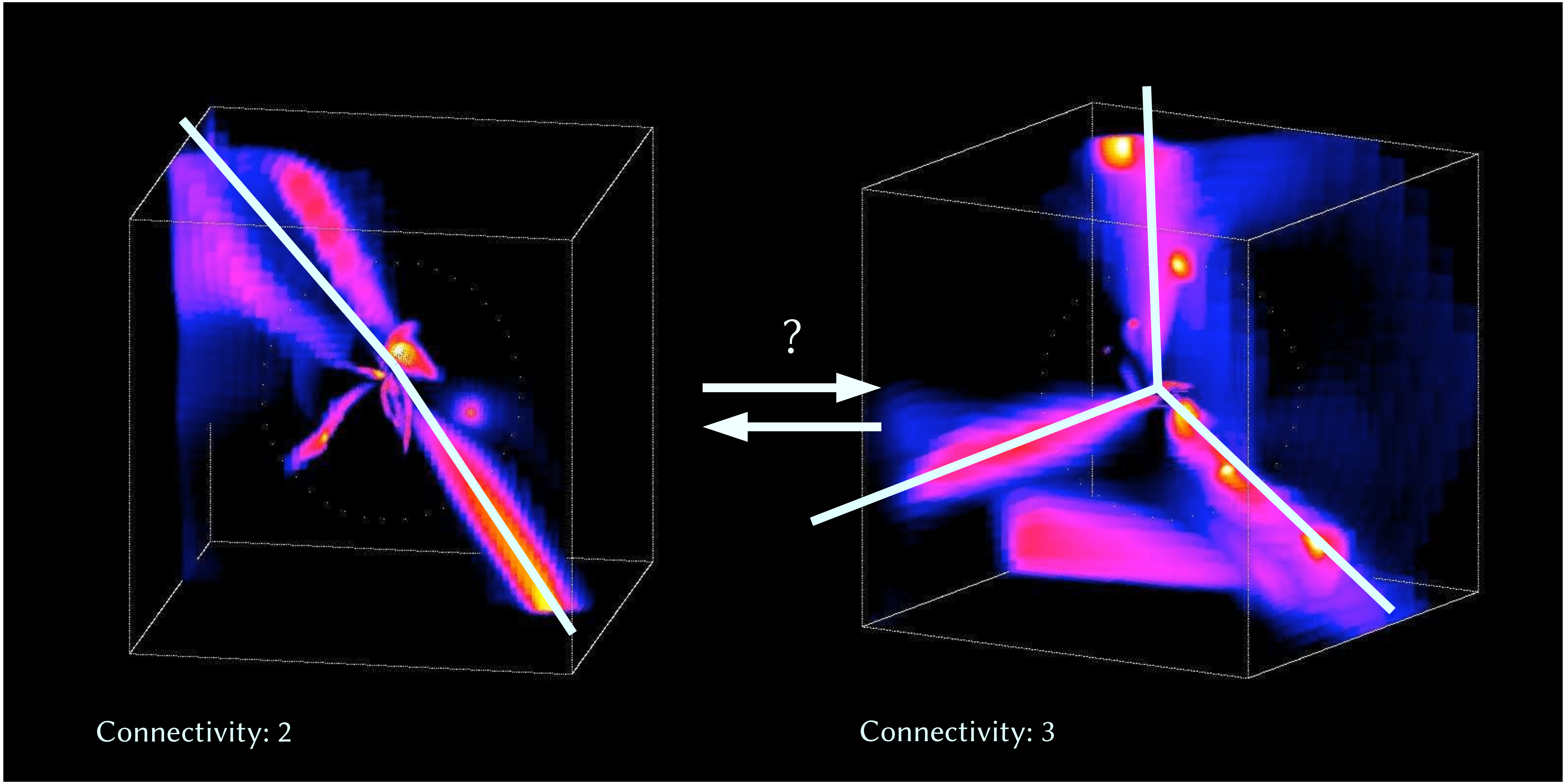

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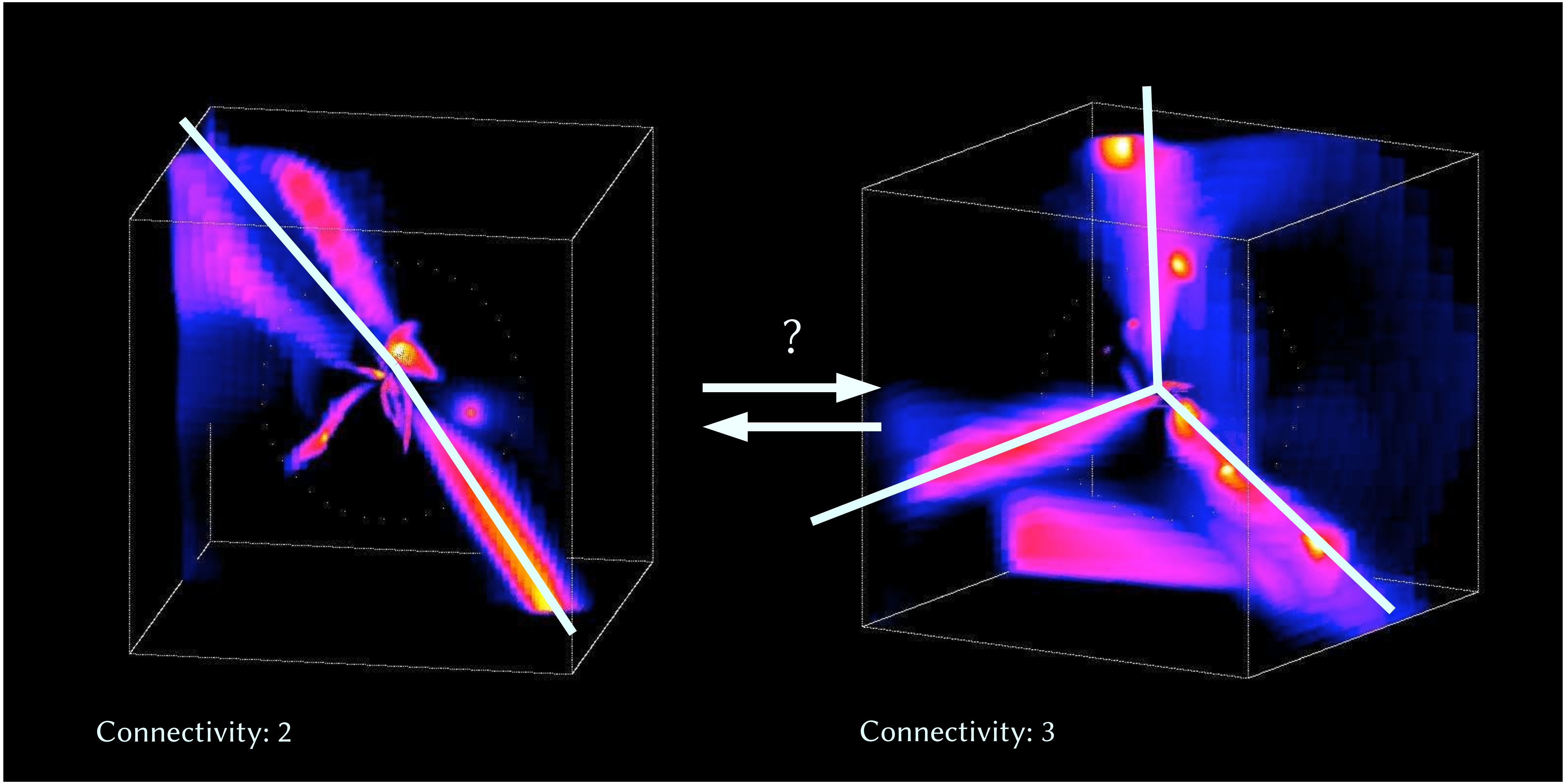
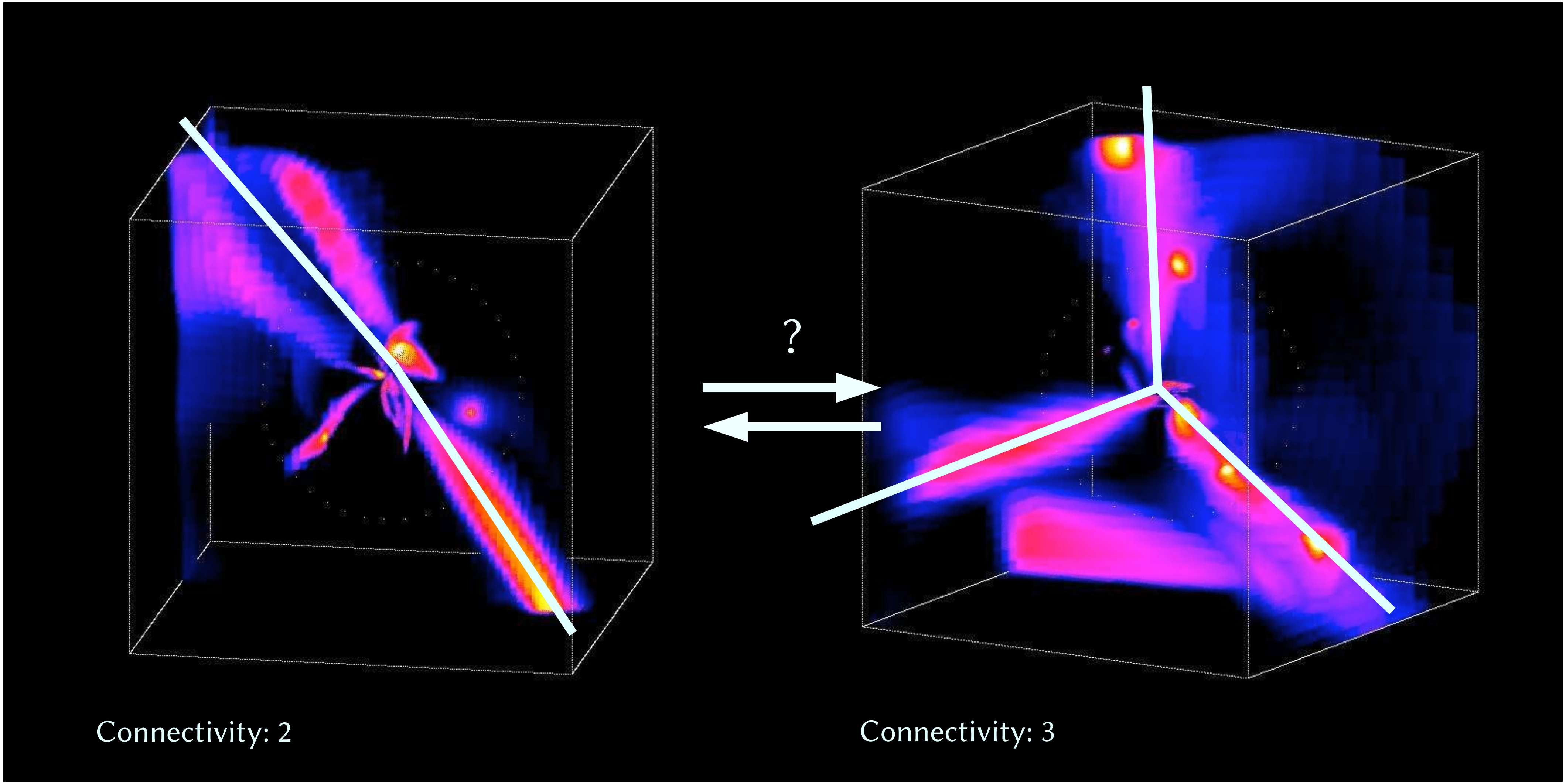


How does **connectivity** evolve with cosmic web? Why 3 filaments?

[Dekel+09; Codis+18 for predictions of the connectivity]

On the problem of connectivity



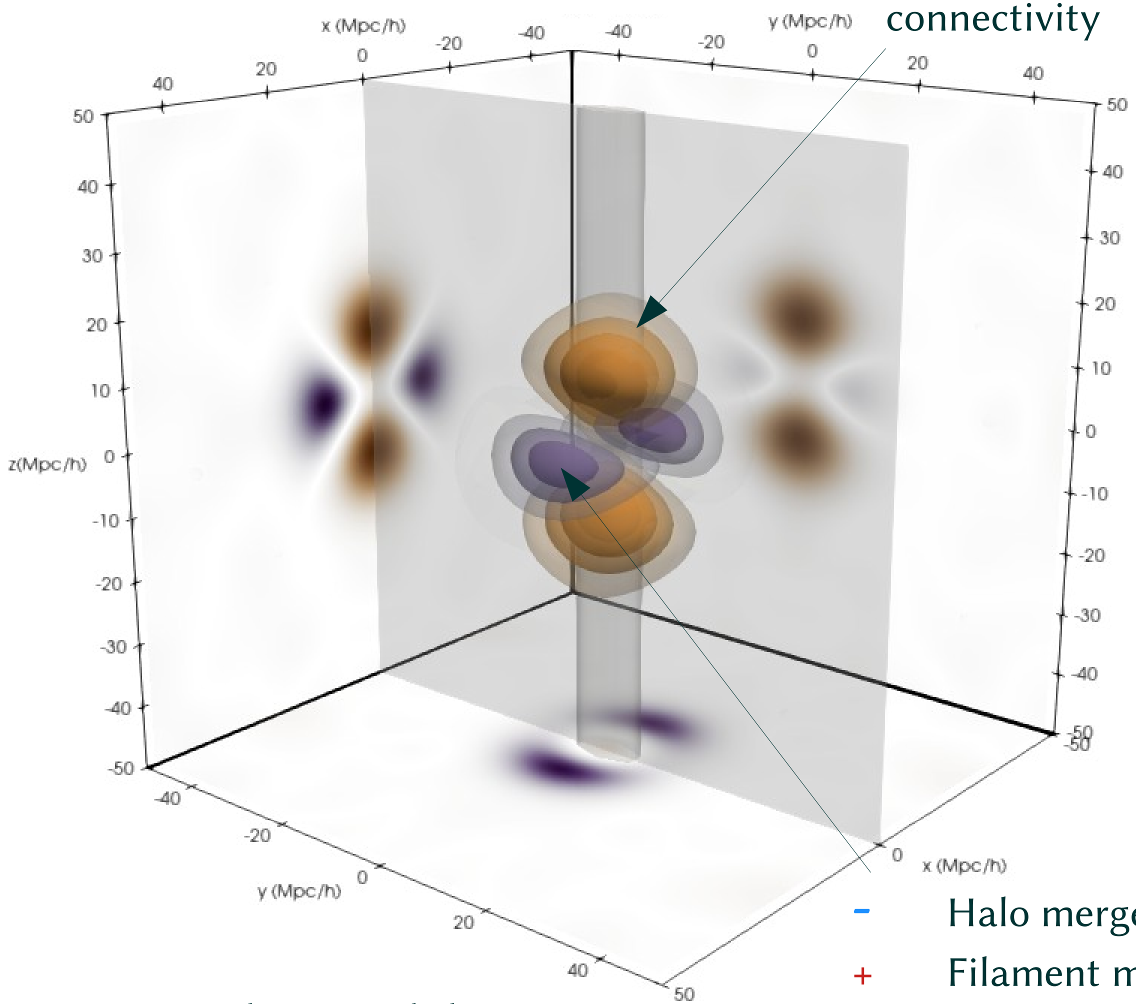
How does **connectivity** evolve with cosmic web? Why 3 filaments?

→ Rely on random realisation + filamentary constrain + numerical estimator

[Dekel+09; Codis+18 for predictions of the connectivity]

Connectivity evolution across the cosmic web

+ Halo mergers
- Filament mergers
⟶ Towards higher connectivity



- Halo mergers
+ Filament mergers
⟶ Towards lower connectivity

Filament-to-halo merger rate, computed numerically. Cadiou+in prep

[Connectivity: Codis+18]

Connectivity evolution across the cosmic web

+ Halo mergers
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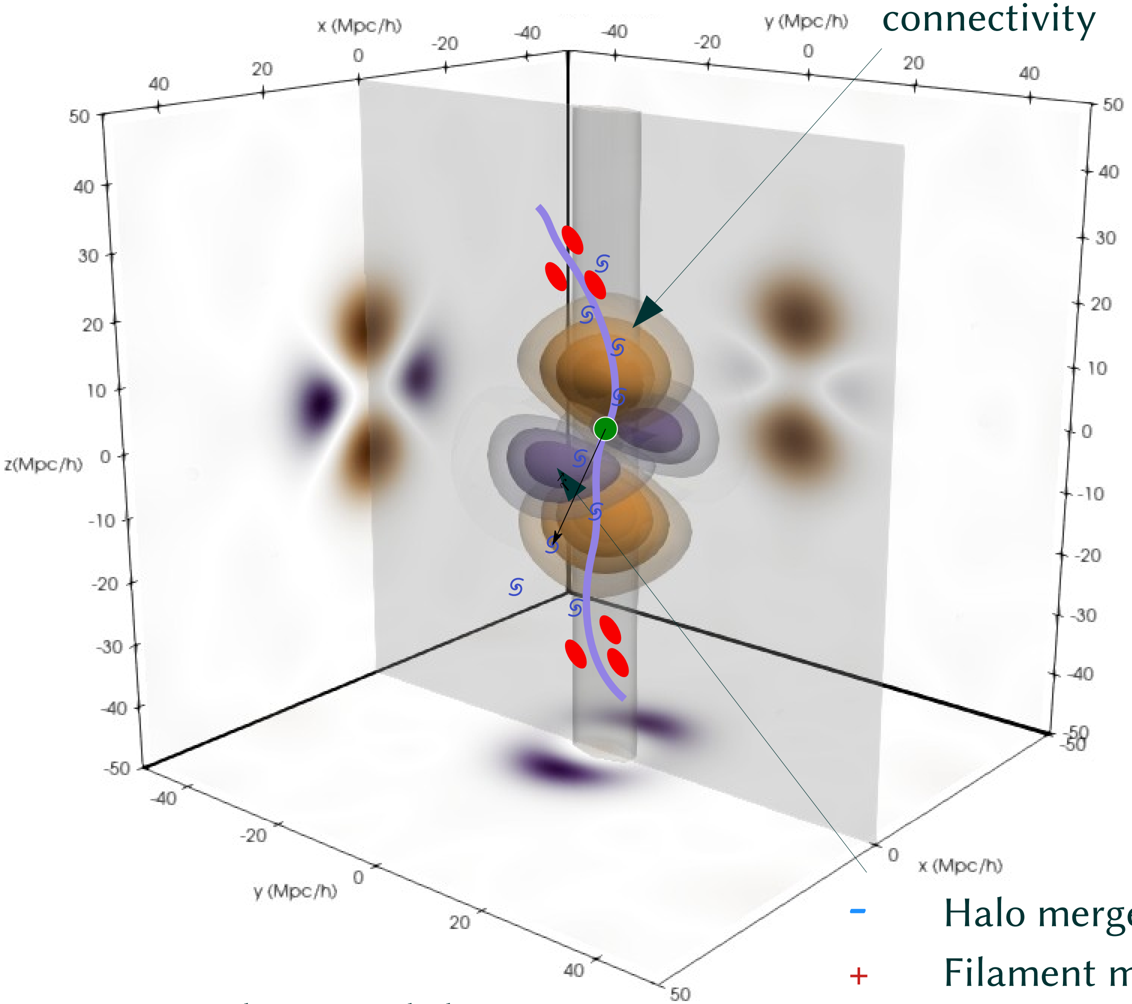

- Halo mergers
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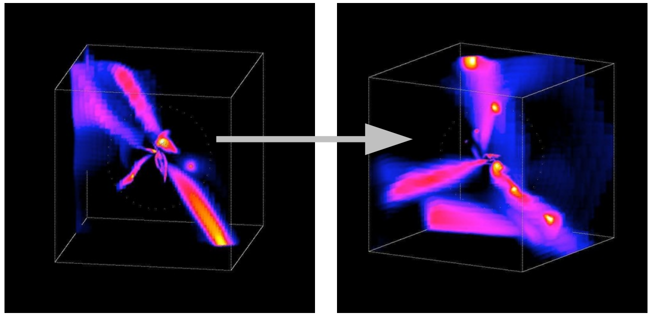

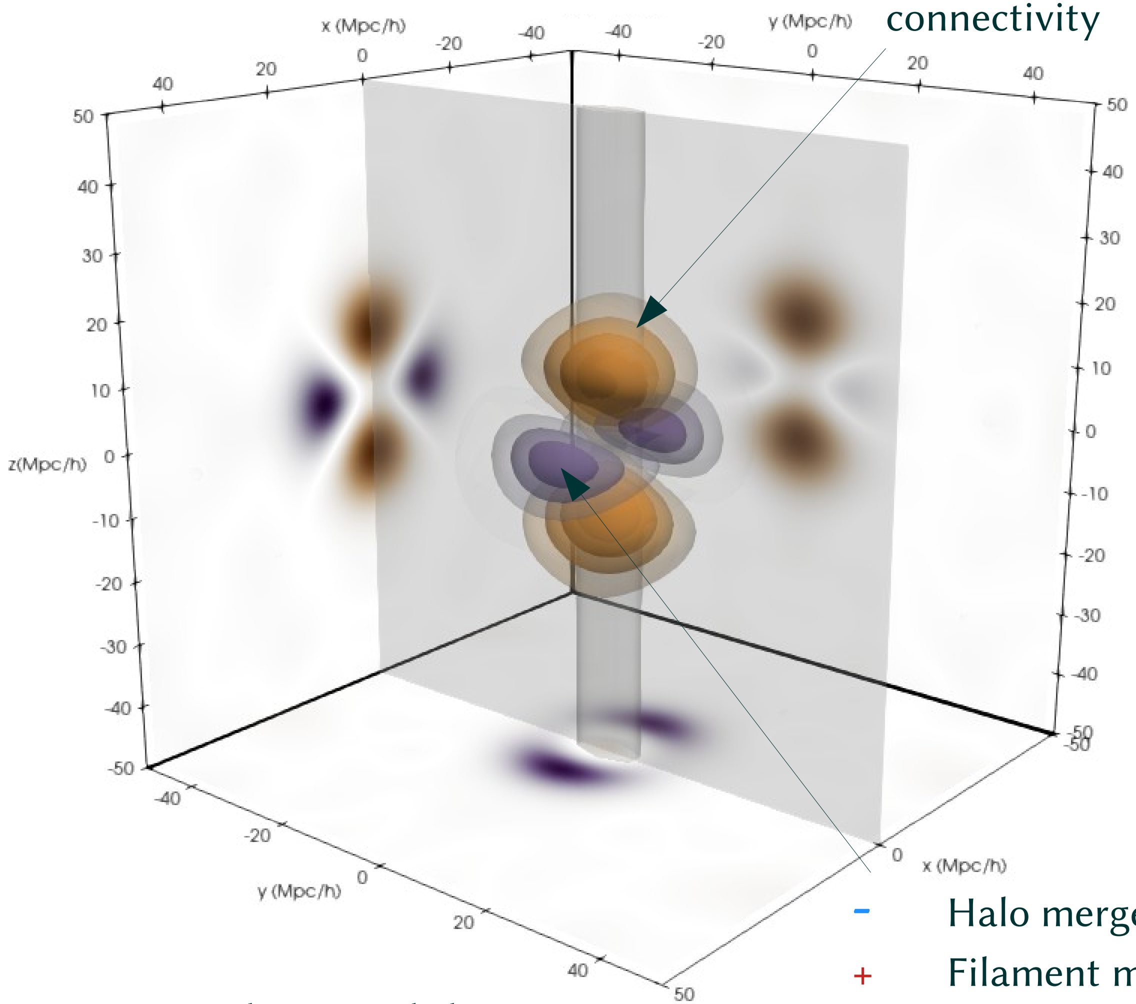

Filament-to-halo merger rate, computed numerically. Cadiou+in prep

- Halo mergers
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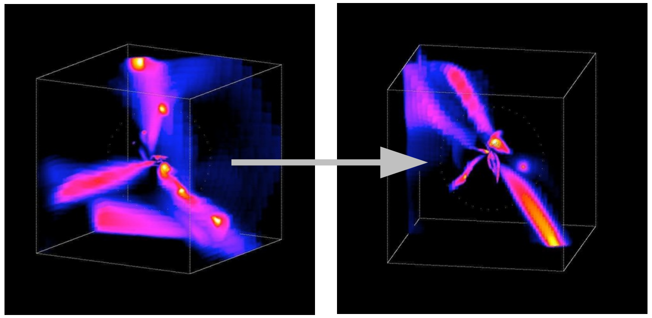


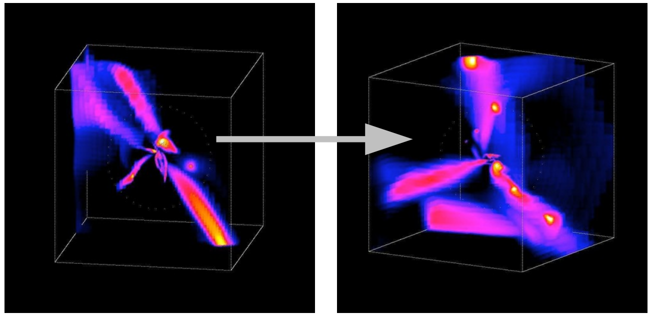
[Connectivity: Codis+18]

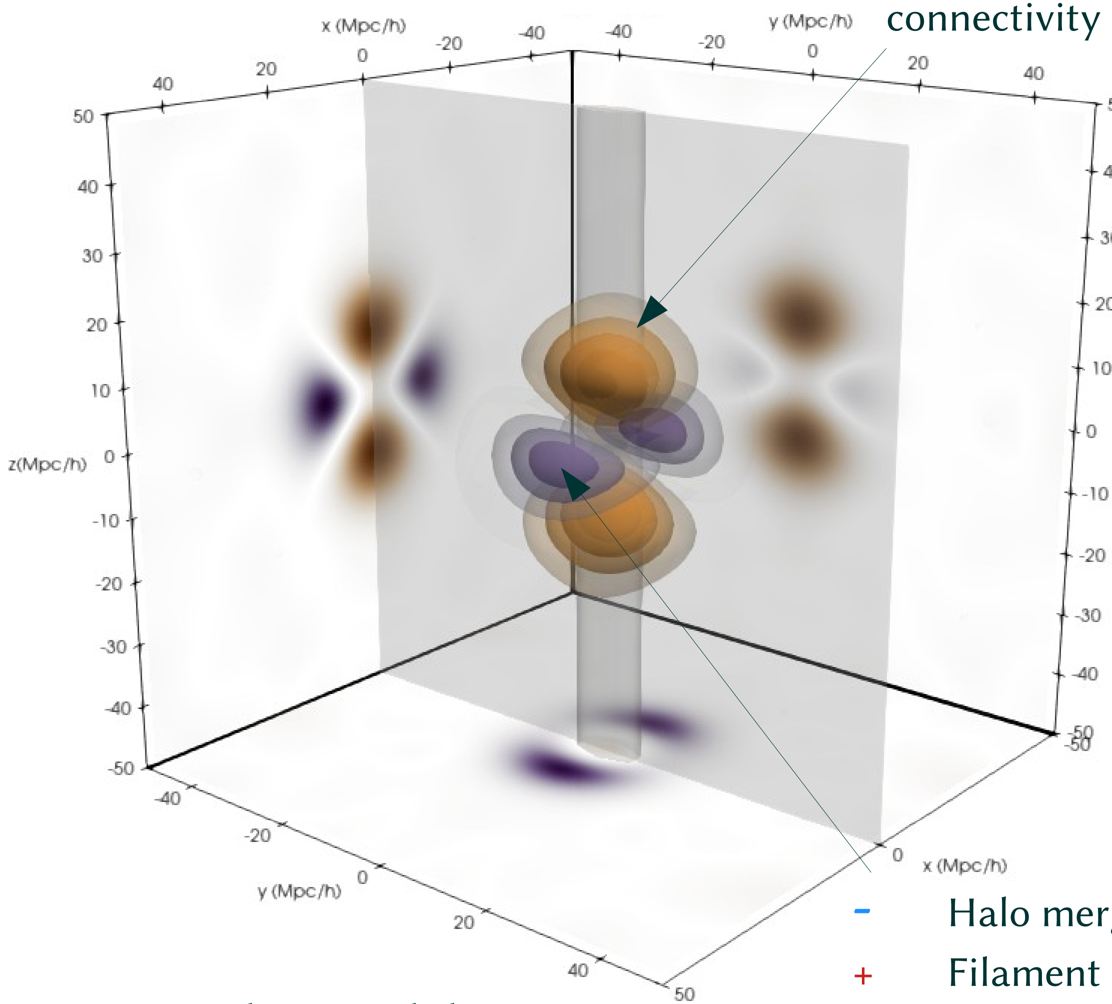
Connectivity evolution across the cosmic web

\+ Halo mergers

\- Filament mergers

⟶ Towards higher connectivity





Filament-to-halo merger rate, computed numerically.
Cadiou+in prep

At **fixed smoothing scale**, in nodes

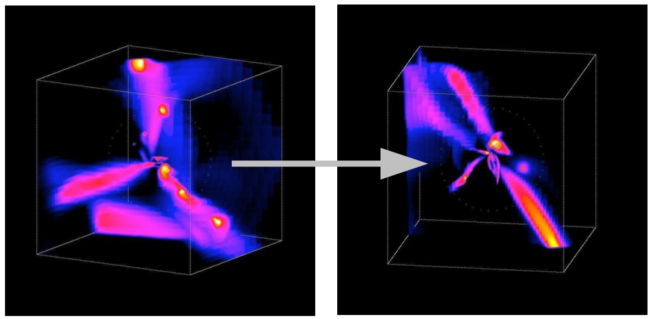
- more halo mergers,
- less filament mergers,
- growing towards higher connectivity,

than in voids.

\- Halo mergers

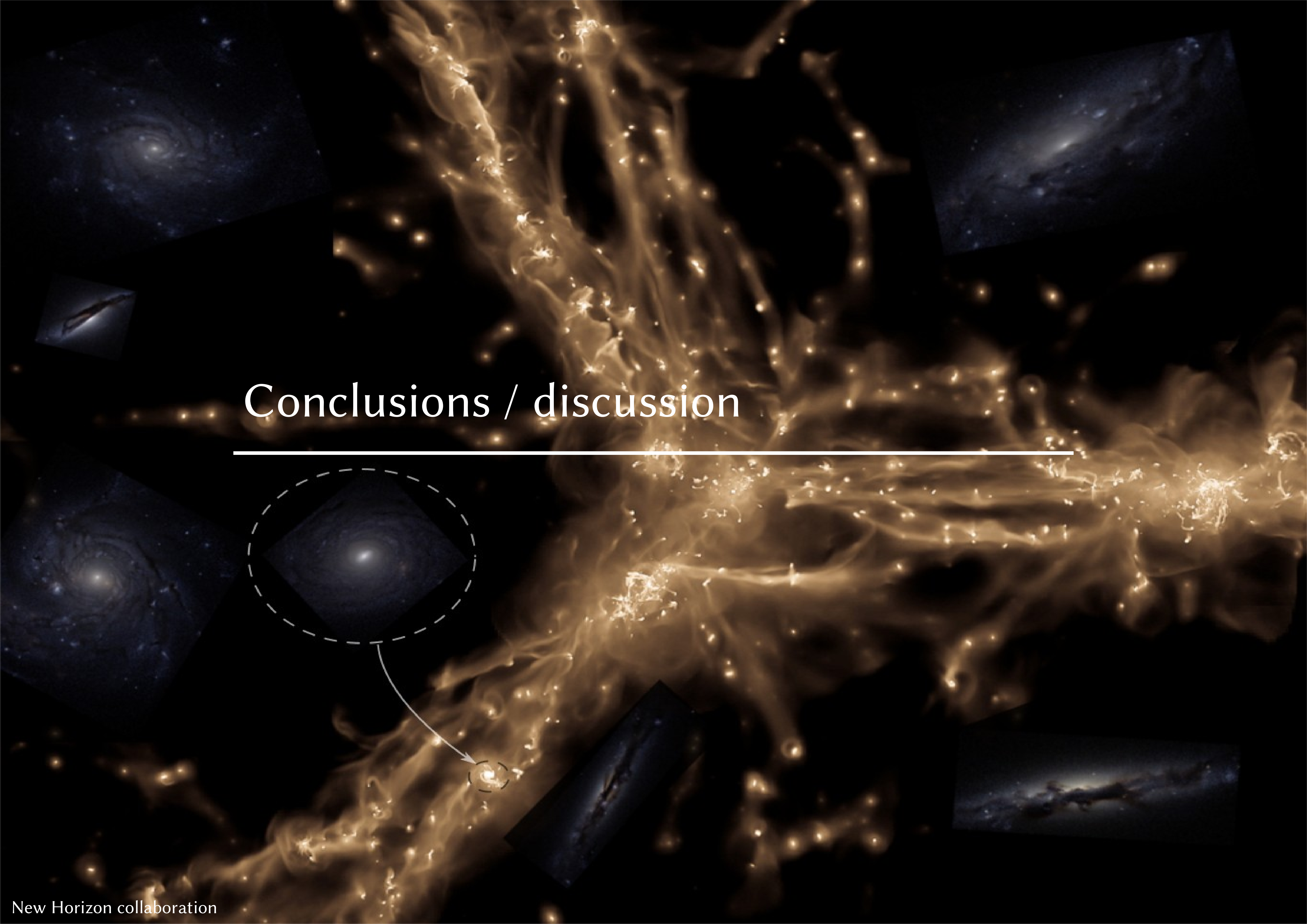
\+ Filament mergers

⟶ Towards lower connectivity



[Connectivity: Codis+18]

Conclusions / discussion

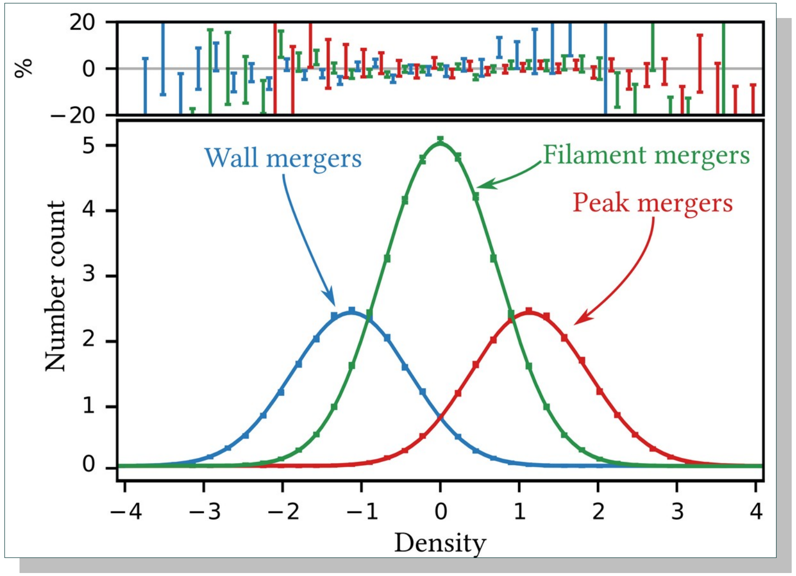


New Horizon collaboration

Conclusion

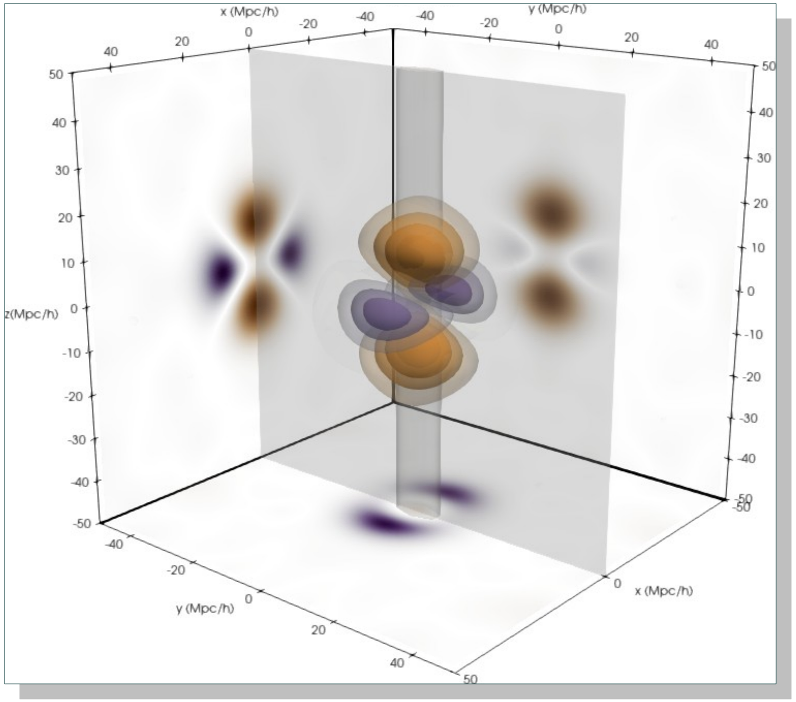
Key points:

- ✔ Describes full change of topology of galactic infall (+ consistent w/ connectivity)
  - ➔ Halo mergers
  - ➔ **Filament mergers + wall (or void) mergers**
- ✔ Very efficient compression
  - ➔ 3D continuous space ⟶ <u>finite</u> set of points in 4D



Achievements

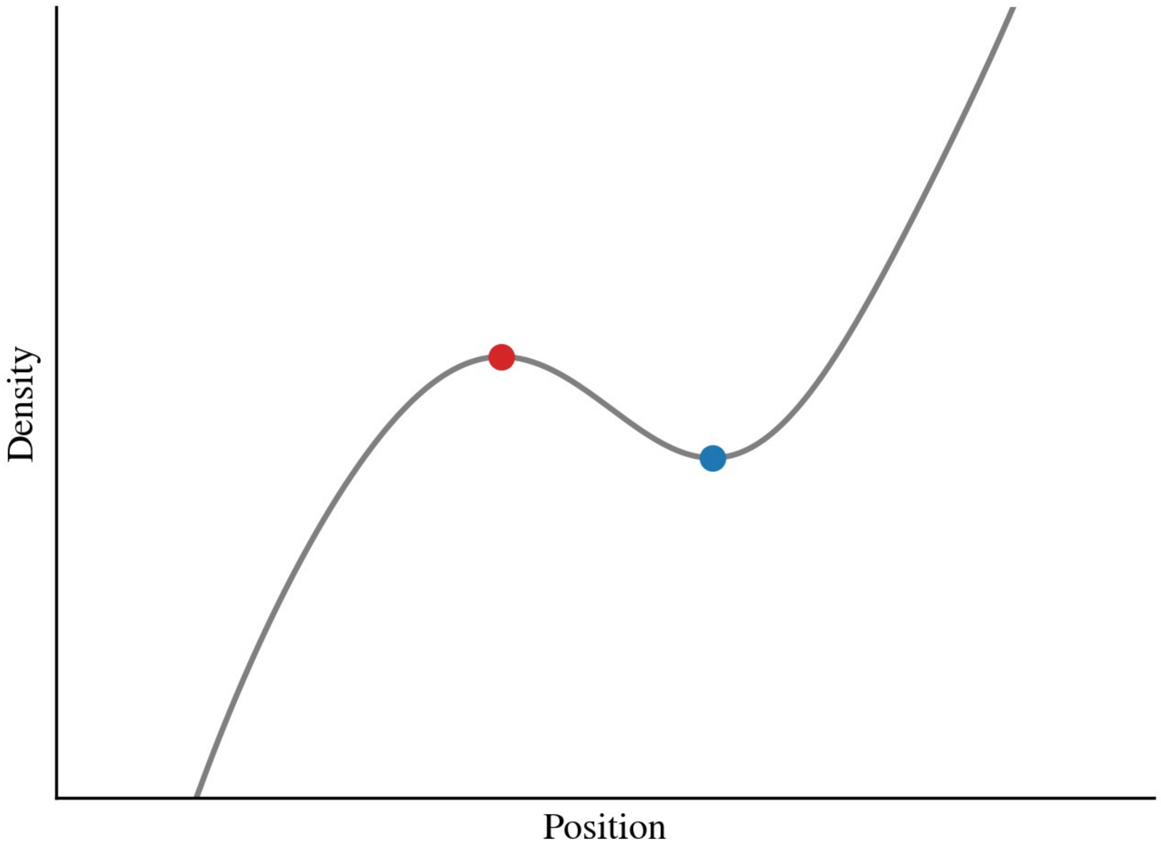
- ✔ Derived theoretical expectations
- ✔ Can be used in numerical simulations
- ✔ Extension to non-linearities (modified gravity or non-linear Universe)
- ✔ Many applications:
  - ✔ Study of assembly bias
  - ✔ Merger rates in mass, time space
  - ✔ Alternative cosmological probe



Future

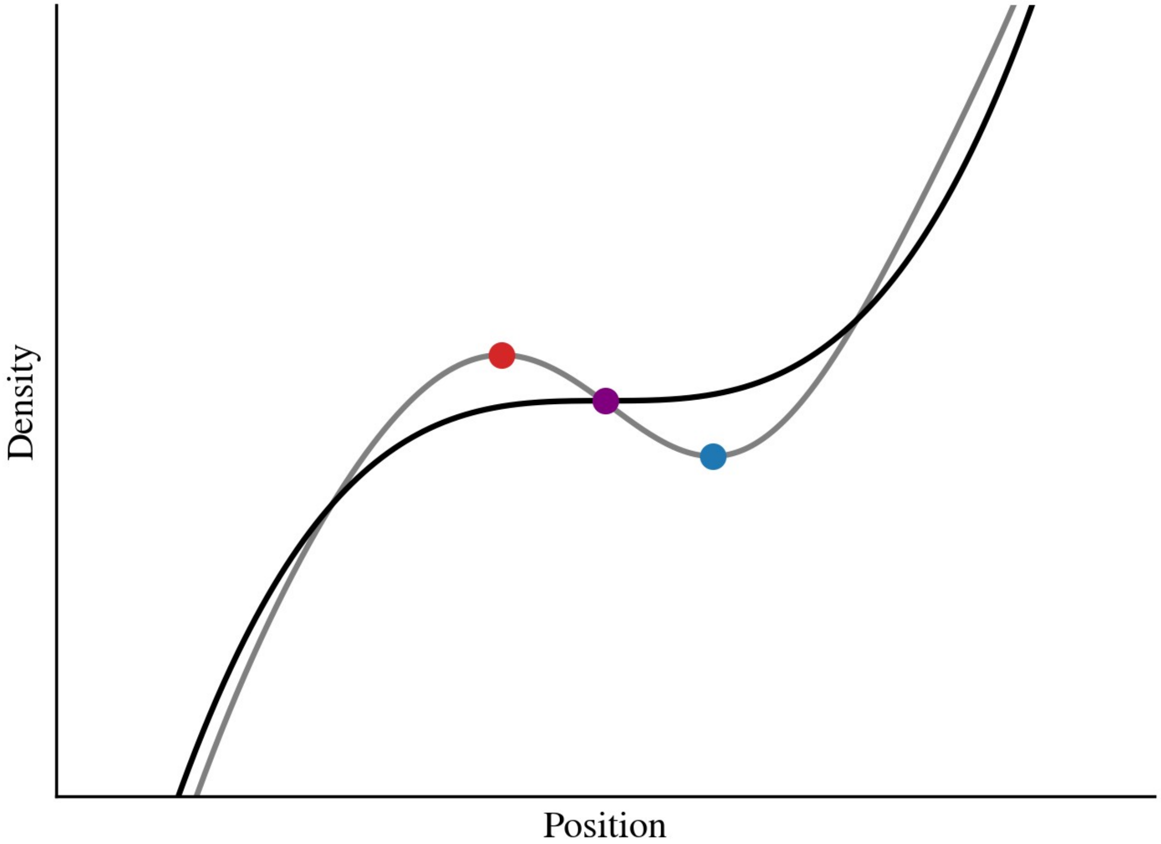
- One-to-one mapping in simulations? Nucleation? Assign mass and time?
- Input to machine learning / halo model

Analytical prediction



[See also Manrique&Salvador 95, 96, Hanami+01]

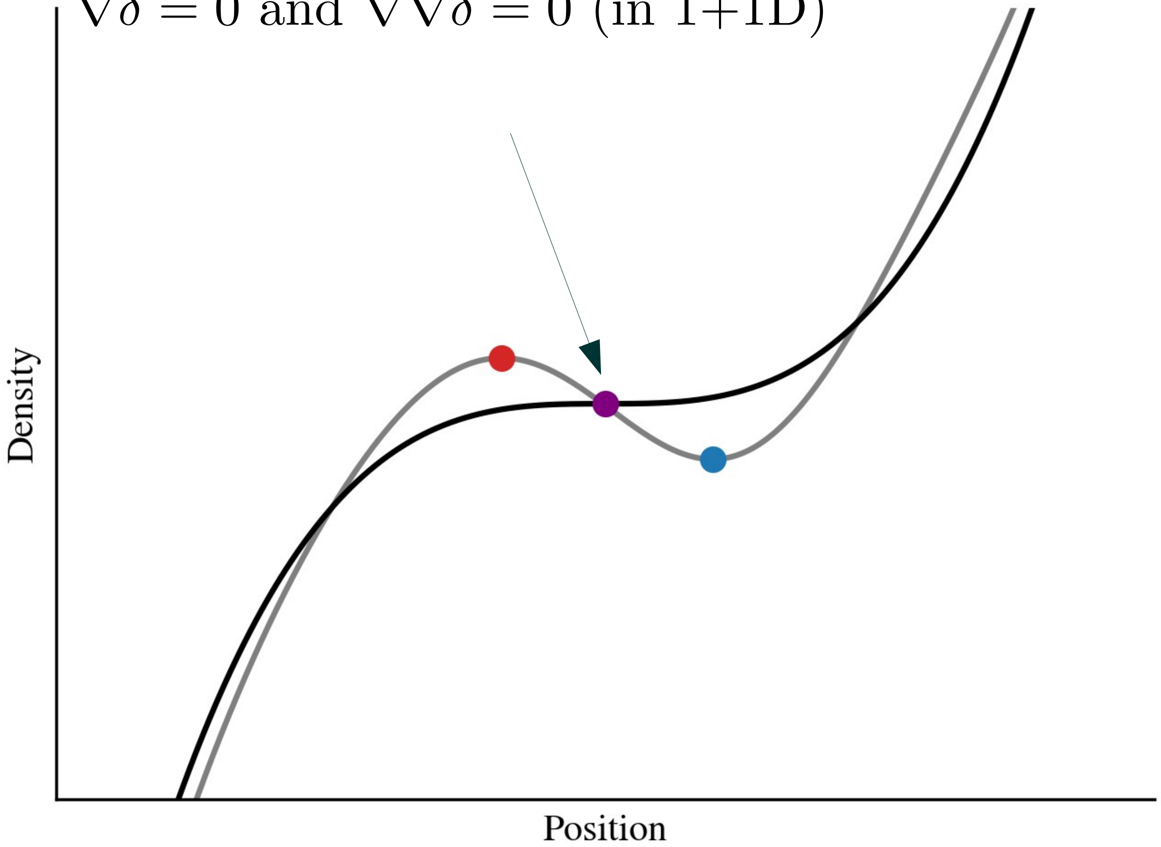
Analytical prediction



[See also Manrique&Salvador 95, 96, Hanami+01]

Analytical prediction

$\nabla\delta = 0$ and $\nabla\nabla\delta = 0$ (in 1+1D)

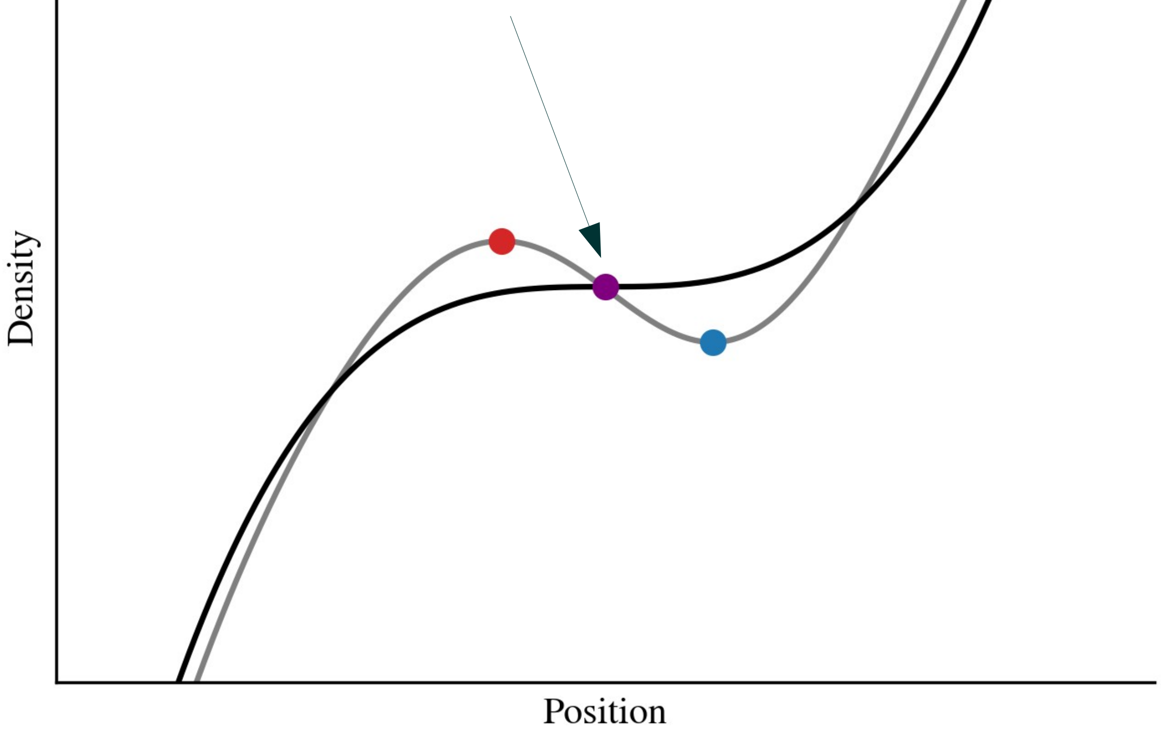


[See also Manrique&Salvador 95, 96, Hanami+01]

Analytical prediction

$\nabla\delta = 0$ and ~~$\nabla\nabla\delta = 0$ (in 1+1D)~~

$\nabla\delta = 0$ and $\det(\nabla\nabla\delta) = 0$ (in 3+1D)



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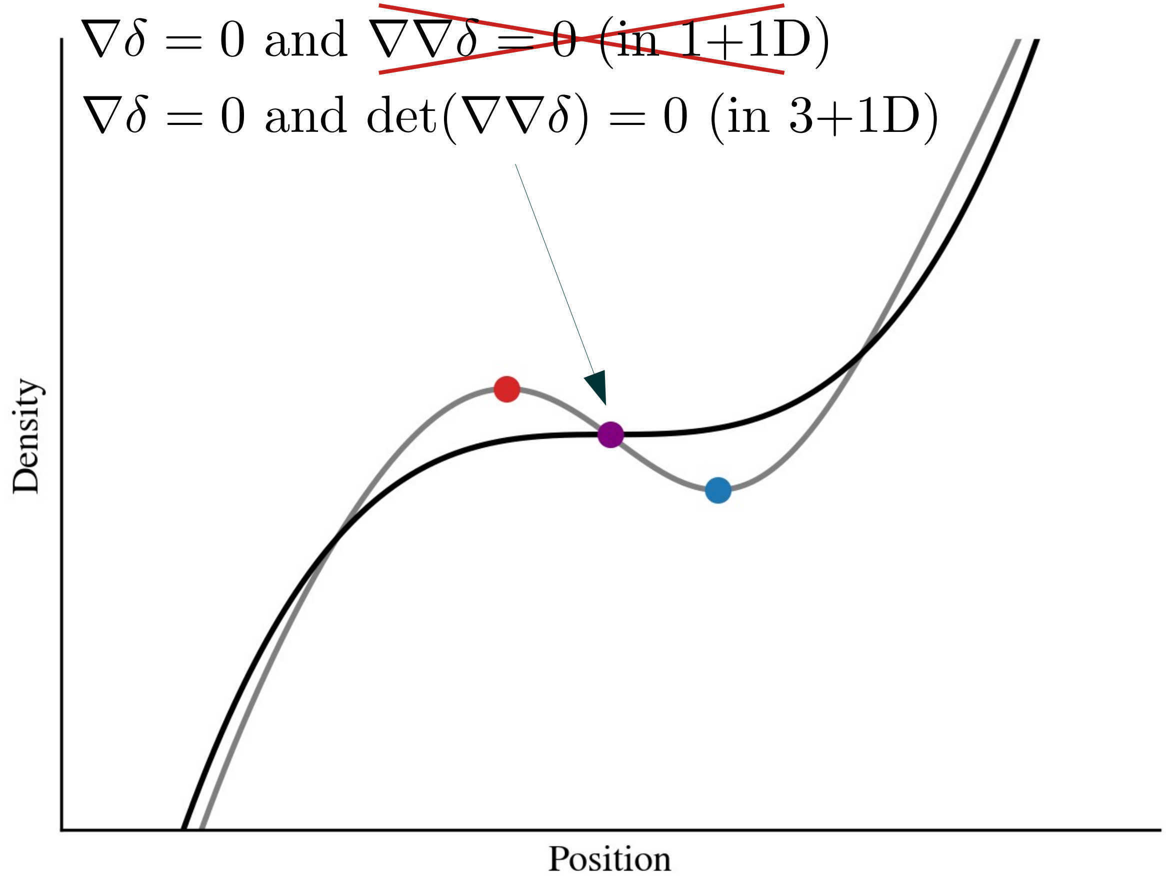
Analytical prediction

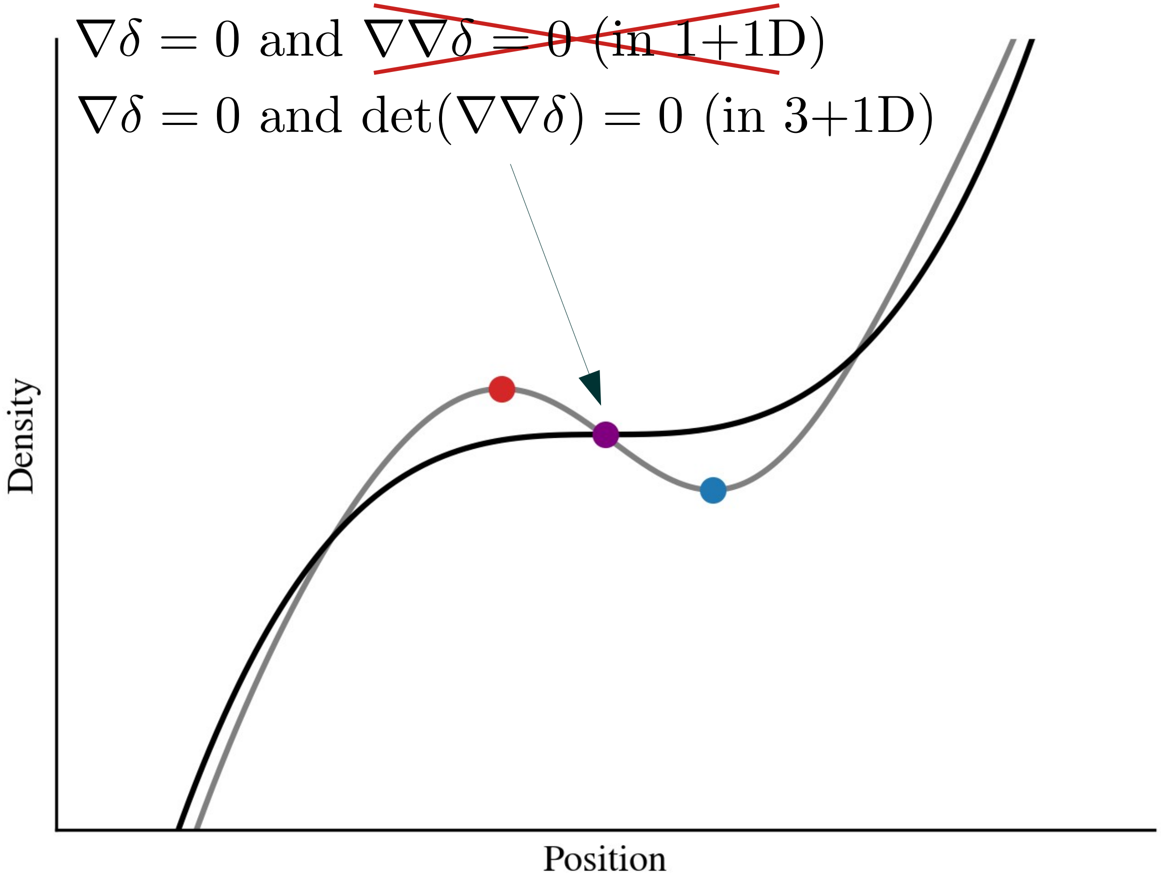


Number count derived from $\mathrm{PDF}(\delta, \nabla\delta, \nabla\nabla\delta, \nabla\nabla\nabla\delta)$

Critical point condition – 10 variables

[See also Manrique&Salvador 95, 96, Hanami+01]

Analytical prediction



Critical *event* condition – 10+10 variables

Number count derived from $\mathrm{PDF}(\delta, \nabla\delta, \nabla\nabla\delta, \nabla\nabla\nabla\delta)$

Critical point condition – 10 variables

[See also Manrique&Salvador 95, 96, Hanami+01]

Analytical prediction

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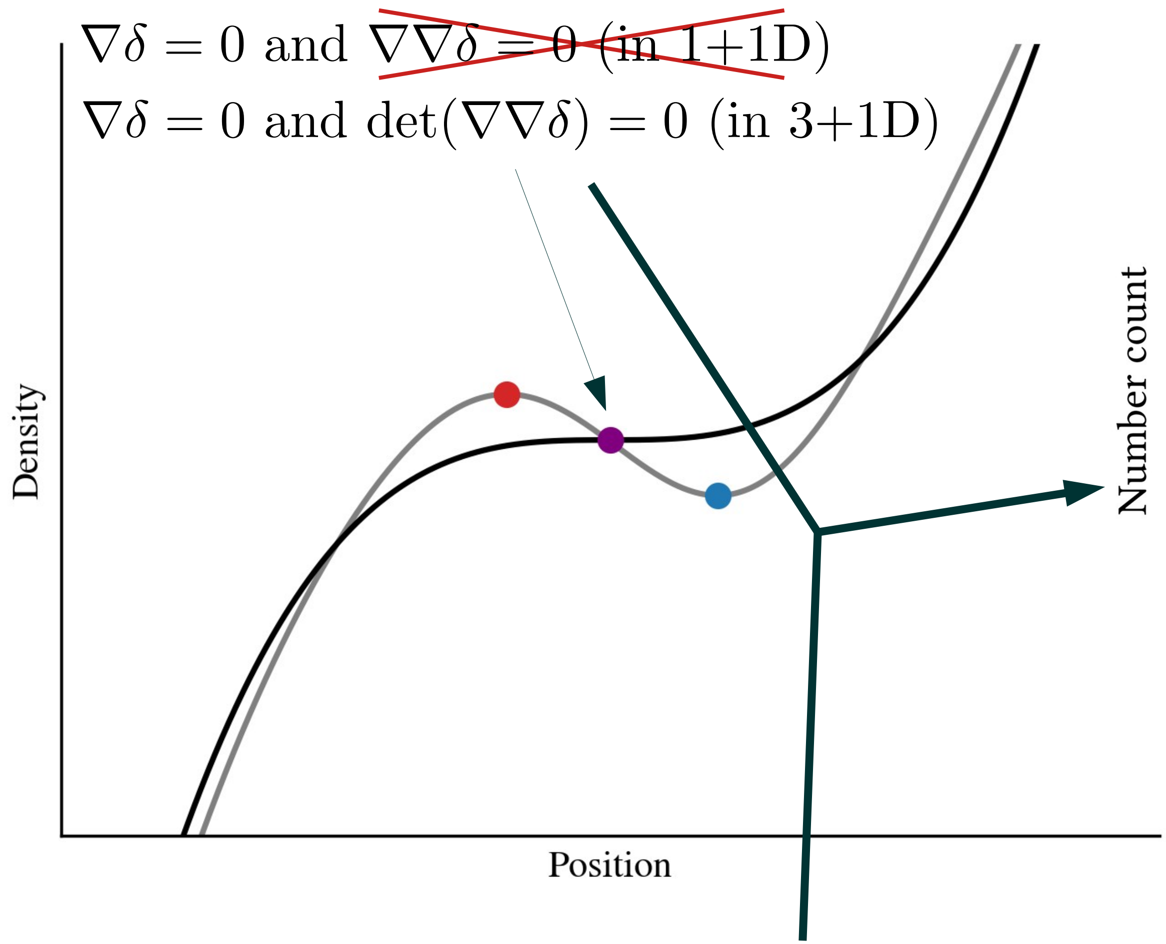

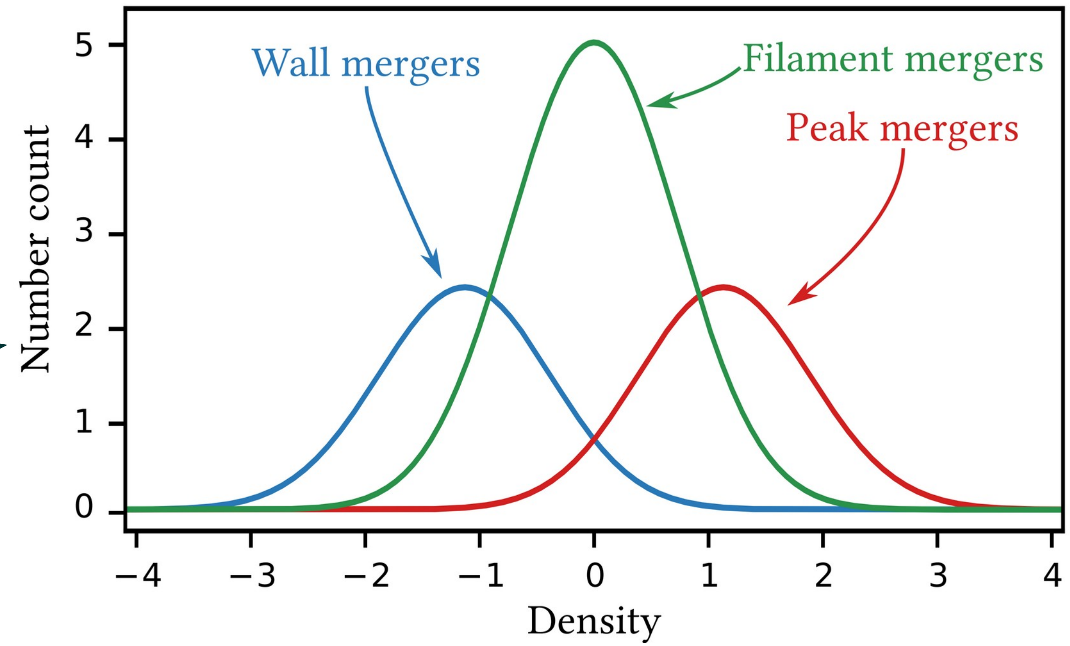

Number count from theory (solid)

Critical *event* condition – 10+10 variables

Number count derived from $\mathrm{PDF}(\delta, \nabla\delta, \nabla\nabla\delta, \nabla\nabla\nabla\delta)$

Critical point condition – 10 variables

[See also Manrique&Salvador 95, 96, Hanami+01]

Analytical prediction

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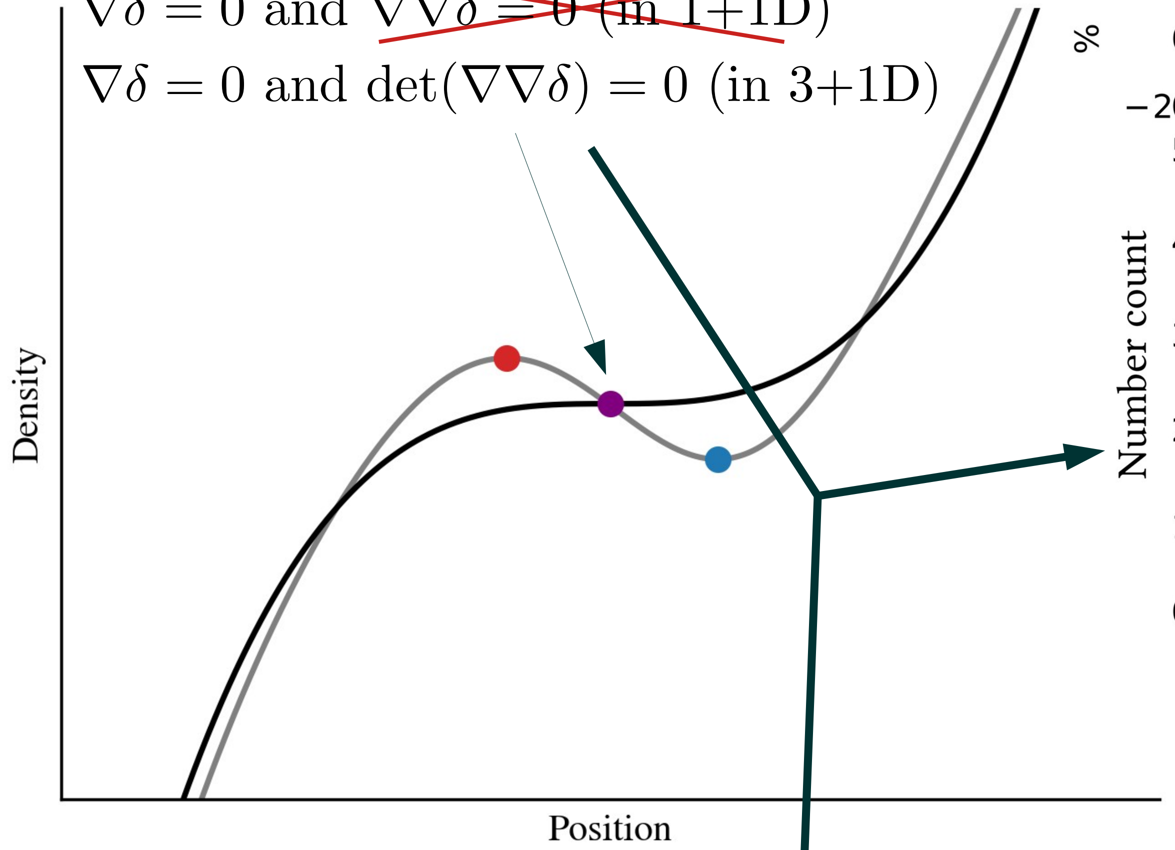
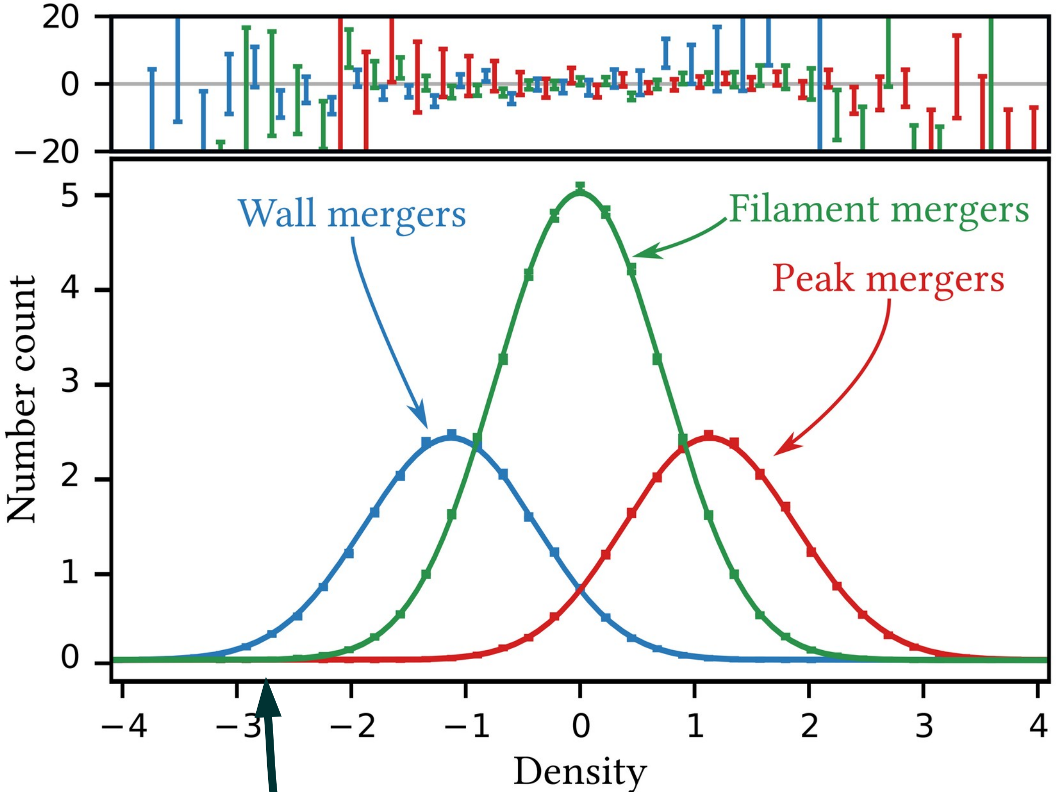
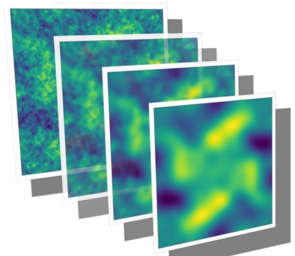




Number count from theory (solid) and numerical (symbols)

Measurements in generated random fields
→ numerical estimator



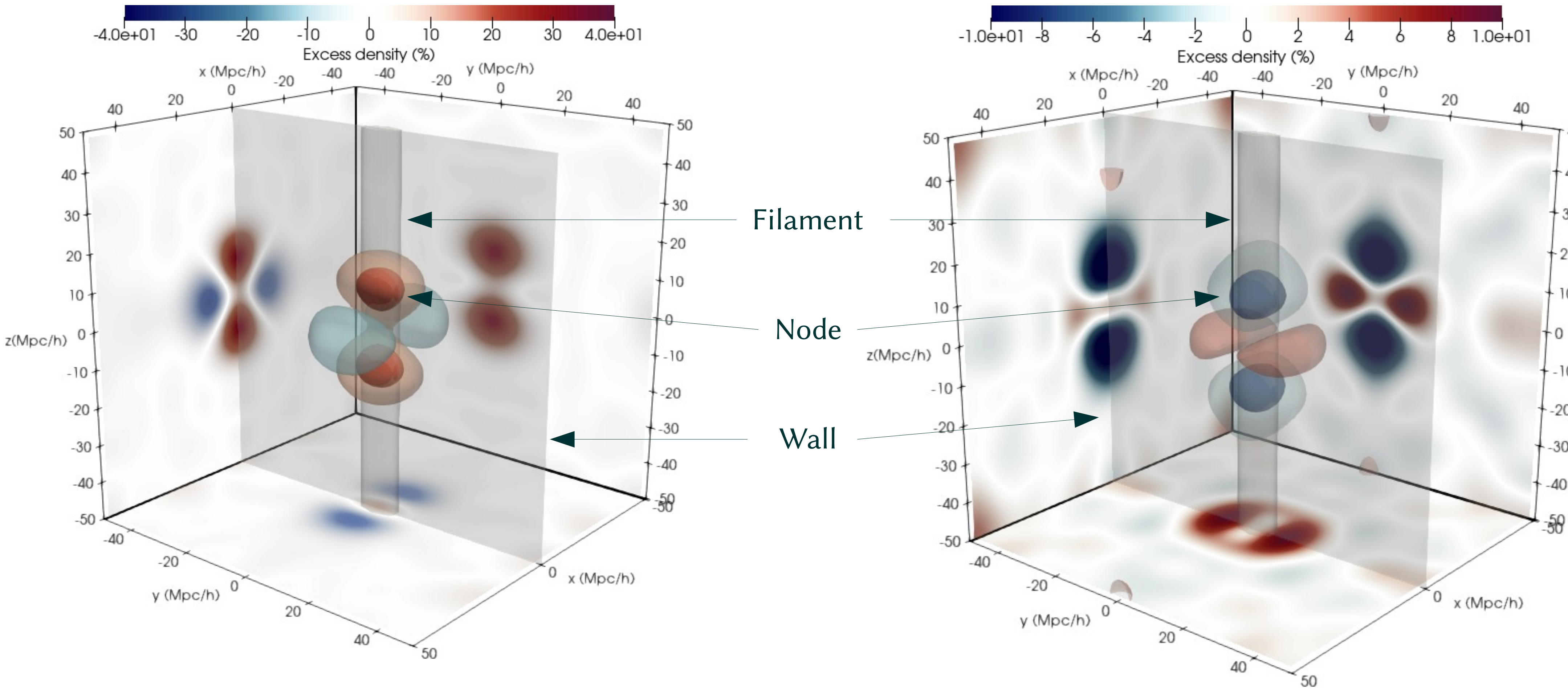
Critical *event* condition – 10+10 variables

Number count derived from $\mathrm{PDF}(\delta, \nabla\delta, \nabla\nabla\delta, \nabla\nabla\nabla\delta)$

Critical point condition – 10 variables

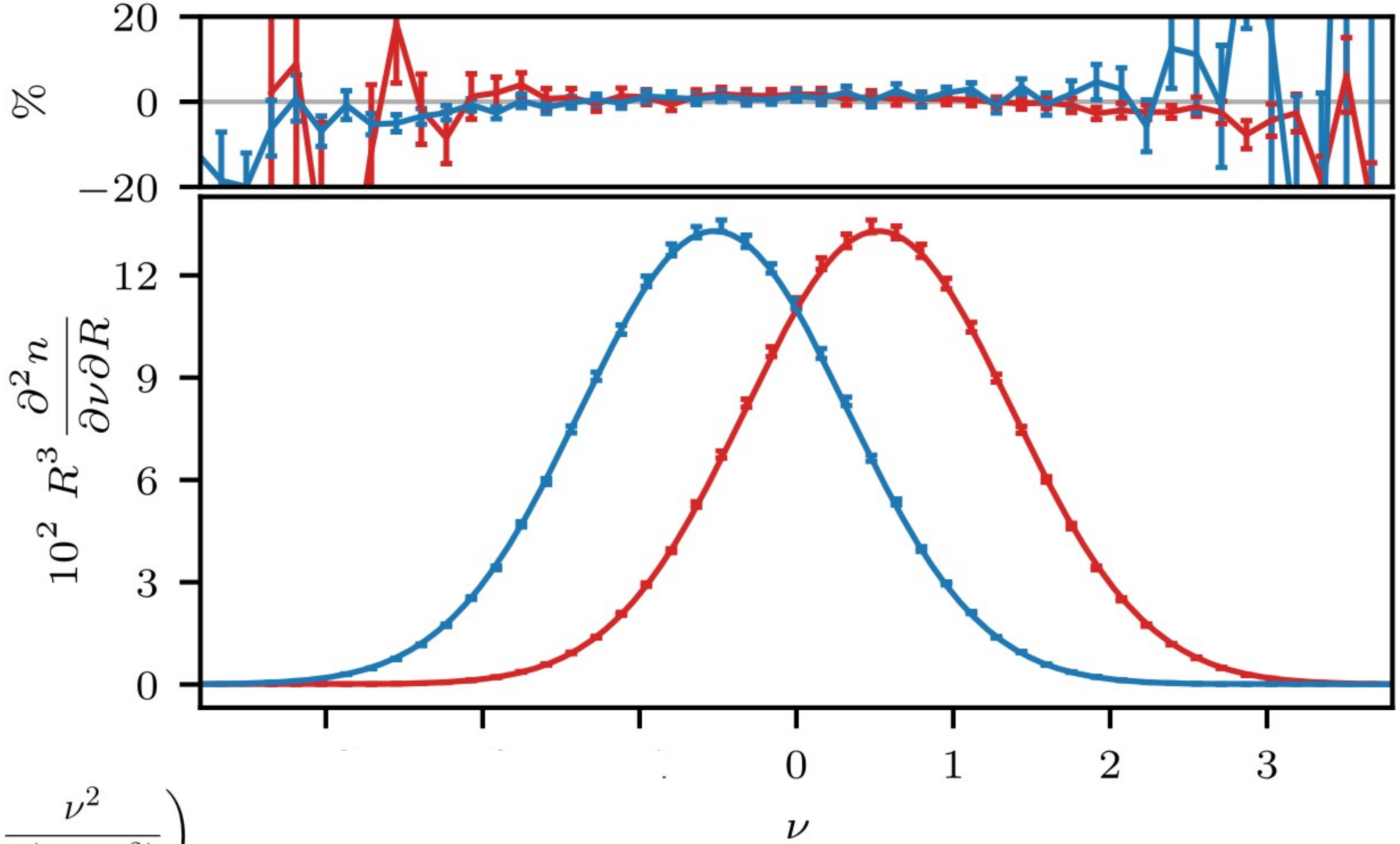
[See also Manrique&Salvador 95, 96, Hanami+01]

Merger rate at **fixed** final mass around filament



Halo merger excess density

Filament merger excess density

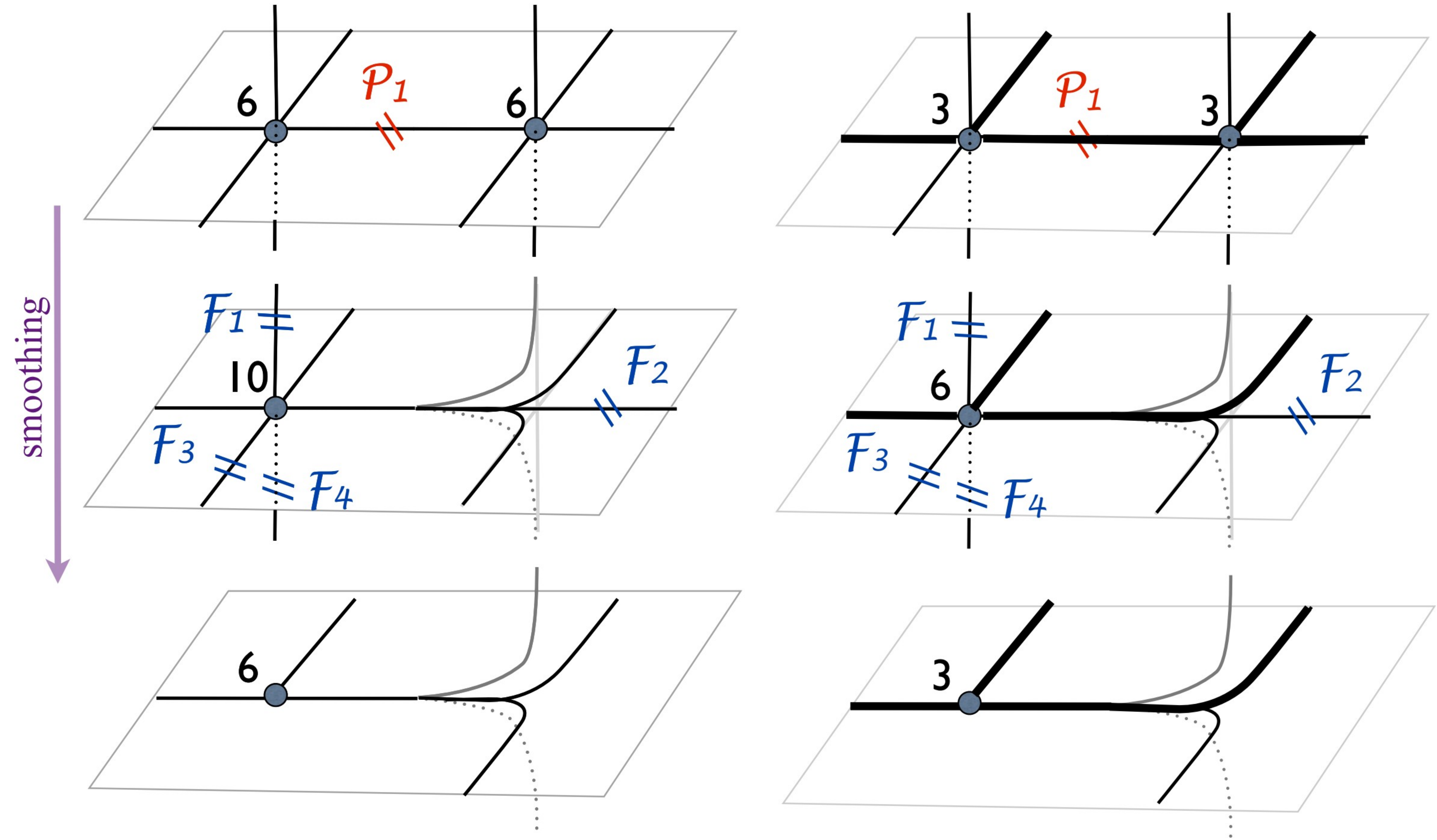


$$\frac{\partial^2 n}{\partial R \partial \nu} = \frac{R C_{\rm odd}}{\tilde{R}^2 R_*^2} \left[\frac{4\gamma\nu\sqrt{1-\gamma^2}}{(3-2\gamma^2)^2} \exp\left(-\frac{\nu^2}{2(1-\gamma^2)}\right) + \frac{\sqrt{8\pi}(2\gamma^4+\gamma^2(\nu^2-5)+3)}{(3-2\gamma^2)^{5/2}} \mathrm{erfc}\left(\frac{-\gamma\nu}{\sqrt{4\gamma^4-10\gamma^2+6}}\right) \exp\left(-\frac{3\nu^2}{6-4\gamma^2}\right)\right],$$

with

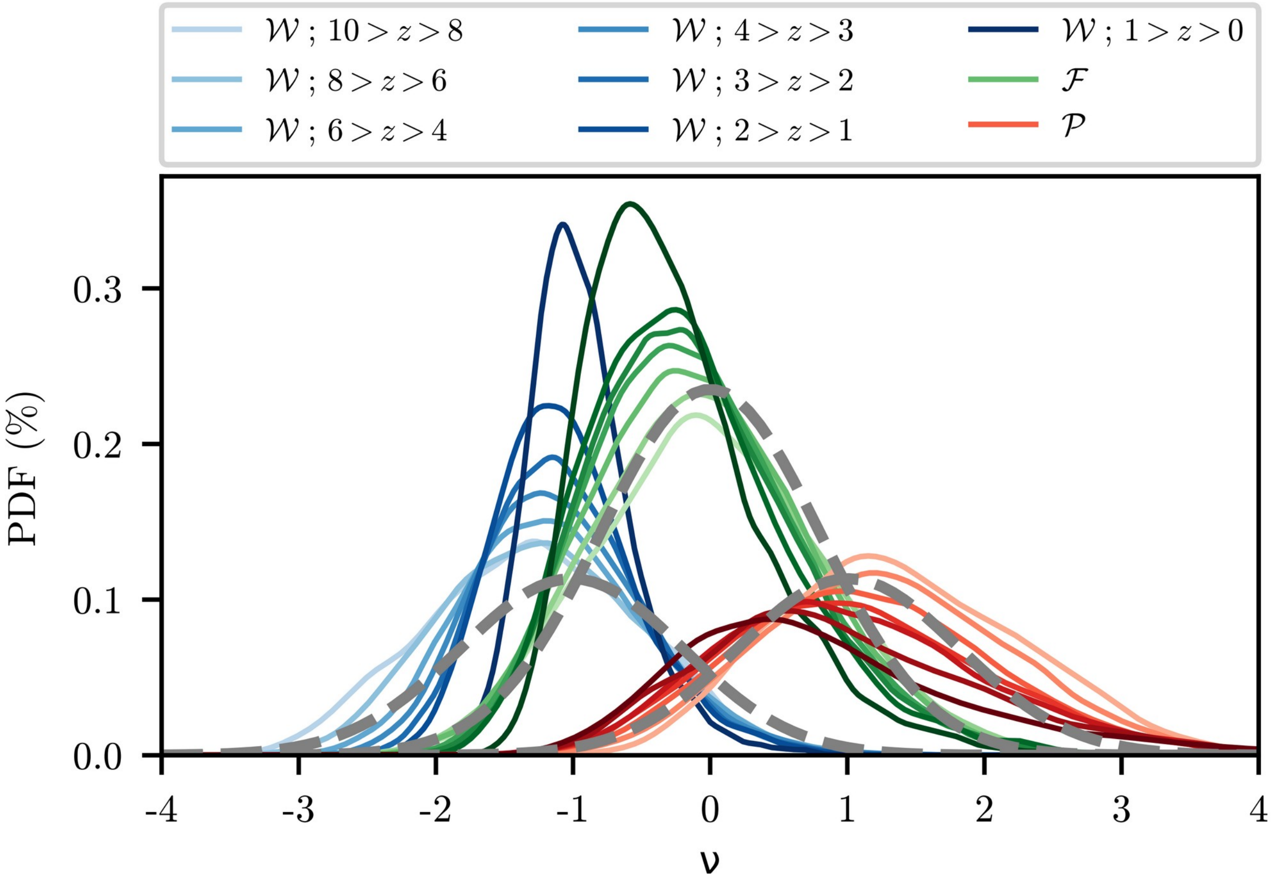
$$C_{\rm odd} = \frac{\hat{\gamma} + 3\hat{\gamma}^2 \tan^{-1}(3\hat{\gamma})}{4\pi^2}, \text{ given } \hat{\gamma} = \sqrt{1-\tilde{\gamma}^2}.$$

Connectivity and critical events – 3+1D case

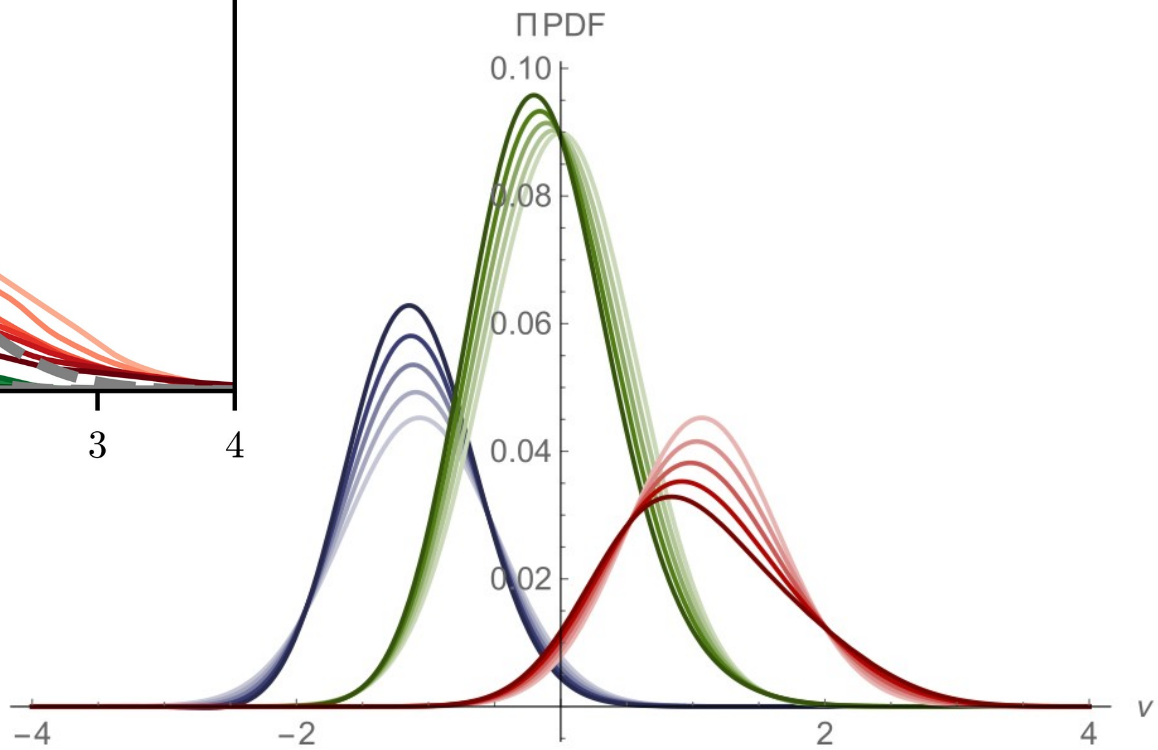


Typical evolution of the connectivity and corresponding critical points.

Comparison with N-body simulations

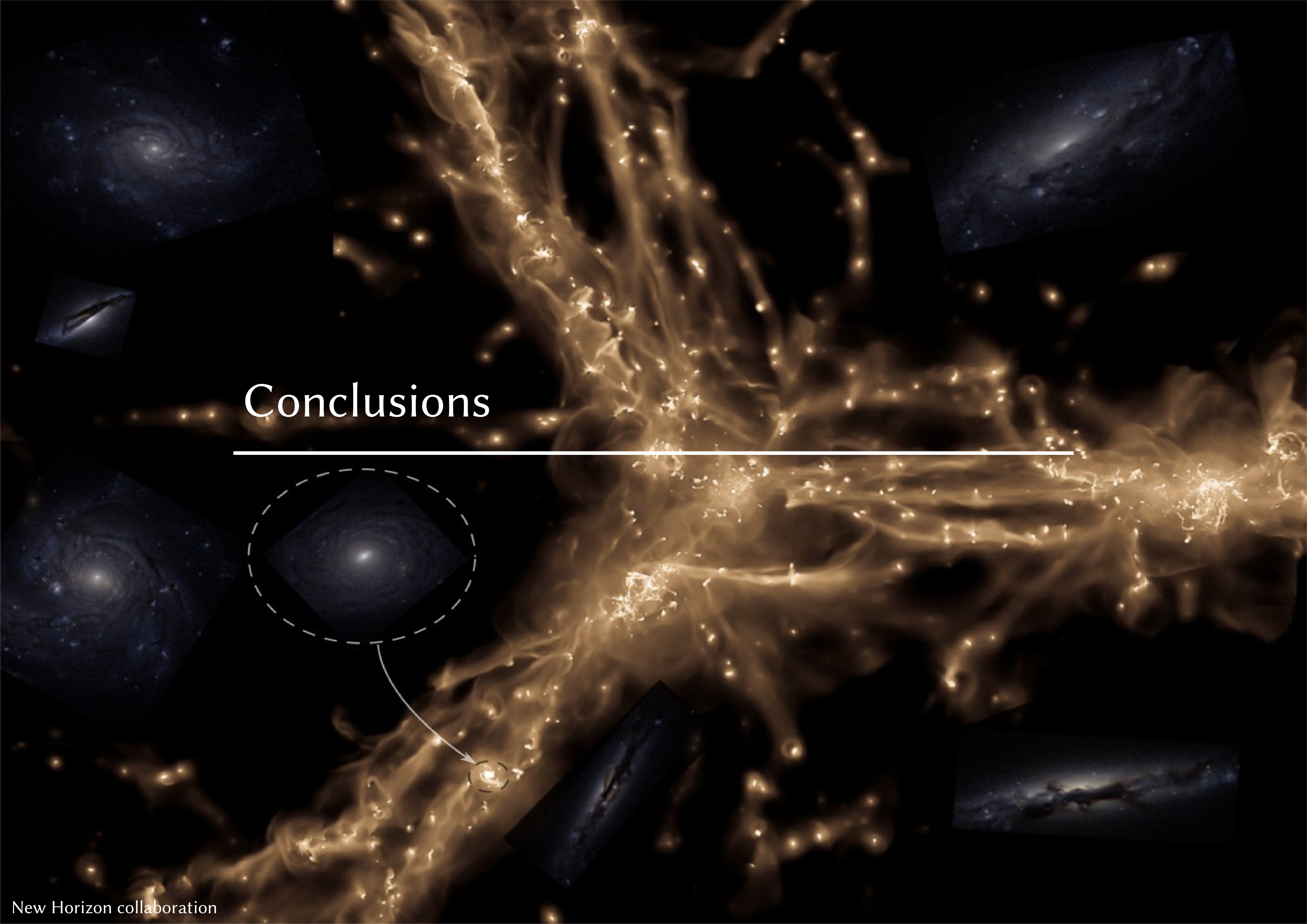


Critical event number counts (solid) in N-body simulations.



Analytical prediction of number counts at first-order in non-gaussianity.

Conclusions



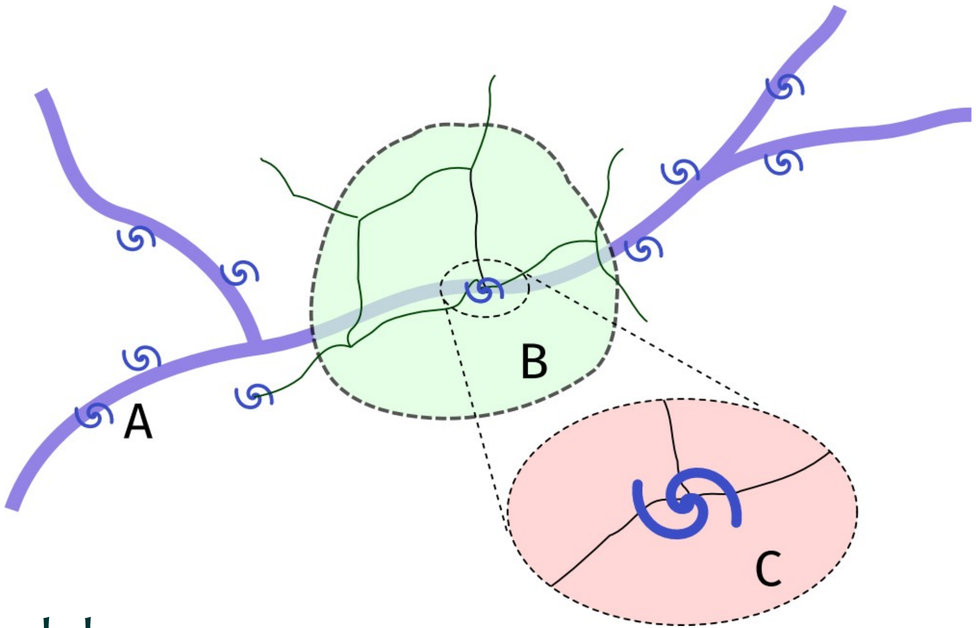
New Horizon collaboration

Cosmic web does **influence** dark matter halo & galaxy formation

- Large-scale filament → explain part of assembly bias signal
- Within Lagrangian patch → growing higher connectivity close to nodes
- Galactic scales → large-scale angular momentum transported to inner regions → gravity-driven

Cosmic web **evolution** best described in terms of

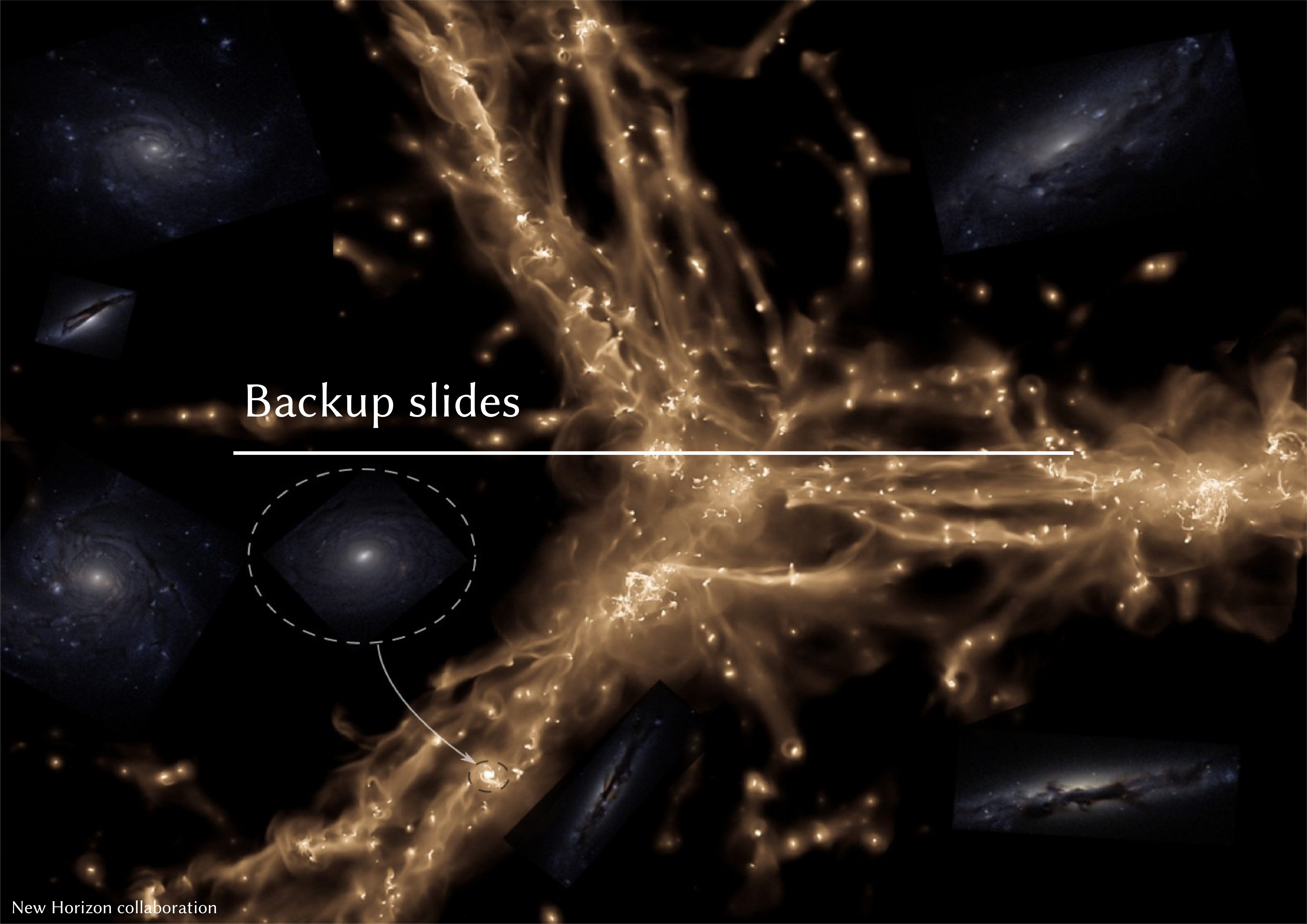
- Critical events:
  → halo mergers,
  → filament mergers,
  → wall mergers.



→ **Anisotropic corrections** on top of classical model

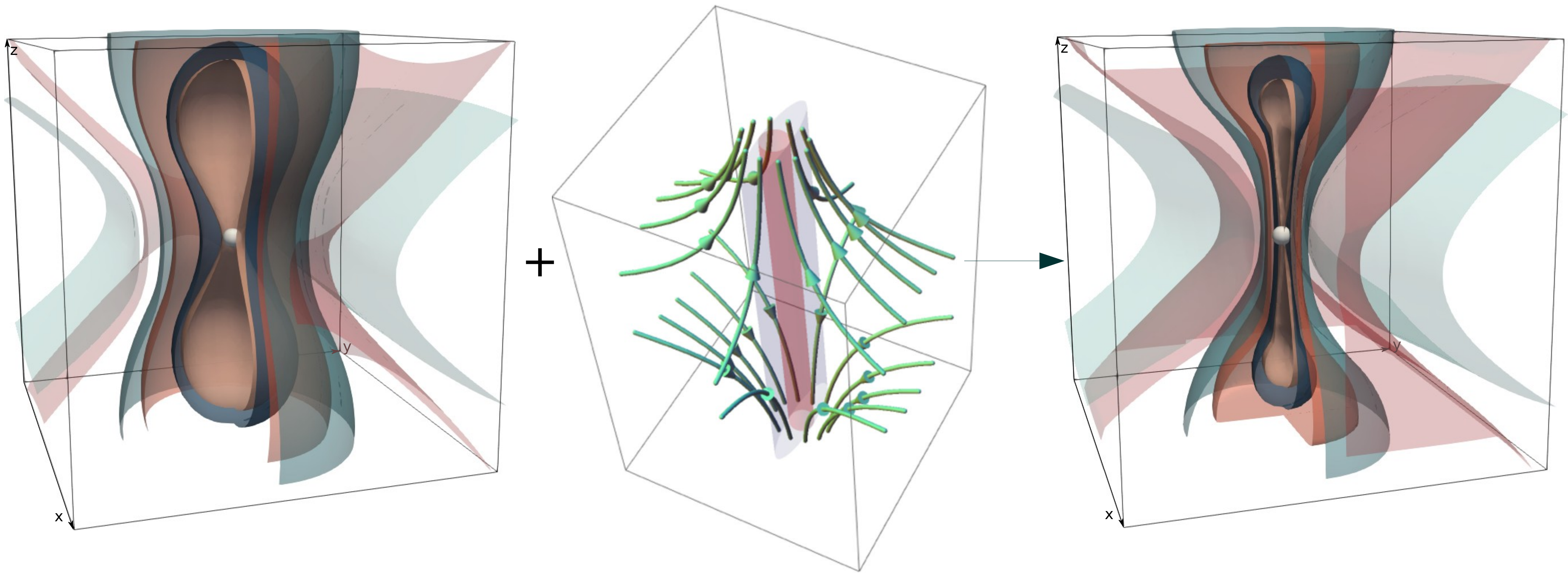
- Tidal interactions ⟶ extend constrained excursion set theory
  - ⟶ *constrained* ellipsoidal collapse?
    [Hahn & Paranjape 14; Ludlow+14; Castorina+16; Ramakrishnan+19]
- Predict galaxy morphology *from initial conditions*
  - ⟶ use augmented merger tree (with filament & wall mergers)?
    [Extending SAMs, see Benson+10 for review]
  - ⟶ use machine learning; critical points as *compression* of information
- Galactic properties
  - ⟶ filament merger ⟹ spin flip *via* cold flows?
  - ⟶ control galactic spin from initial conditions?
    [Roth+16; Rey&Pontzen 17]
  - ⟶ control AGN activity from initial conditions?
    [Porqueres+18; Man+19; Huang+19]

Backup slides

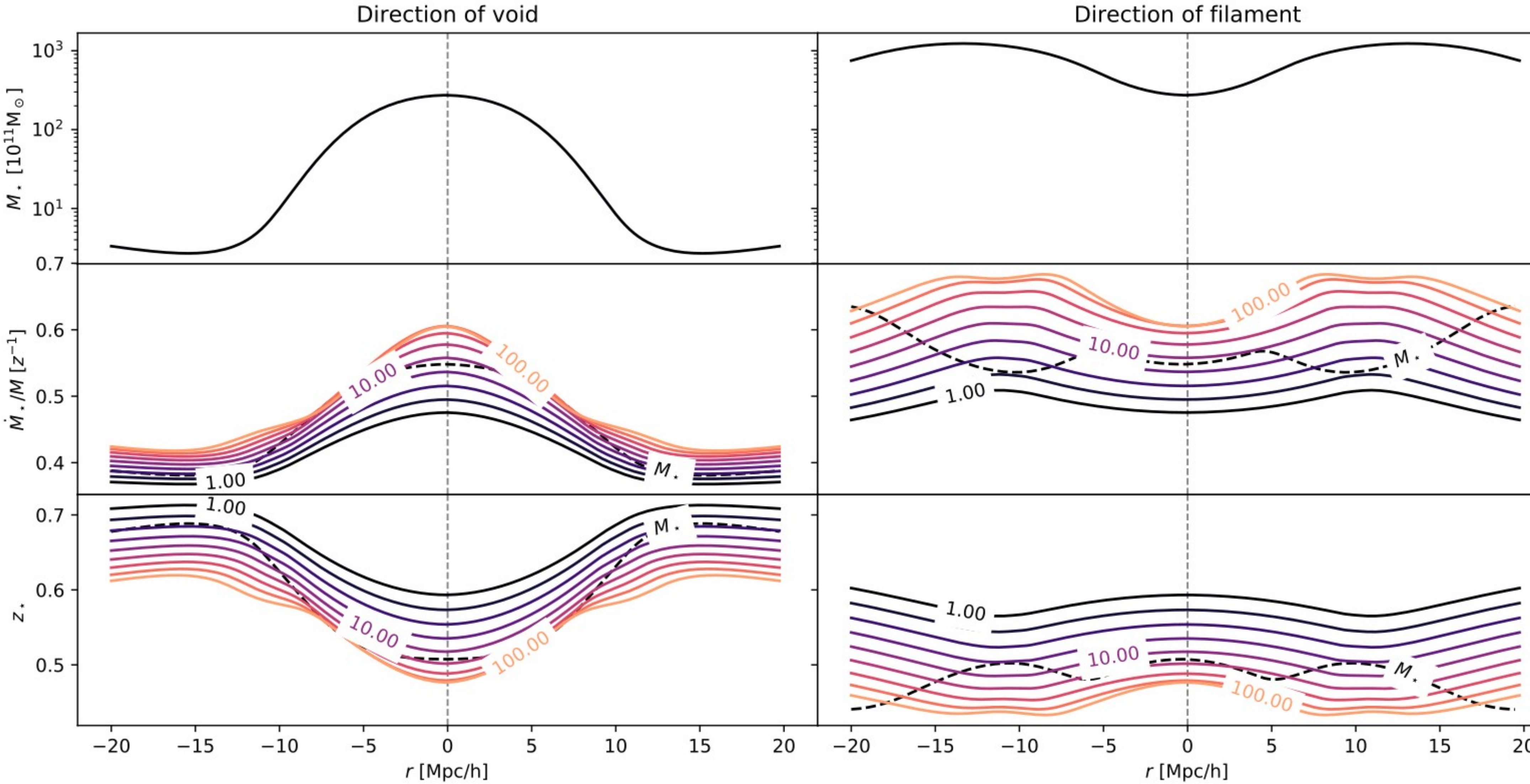


New Horizon collaboration

Accounting for Zel'dovich displacement

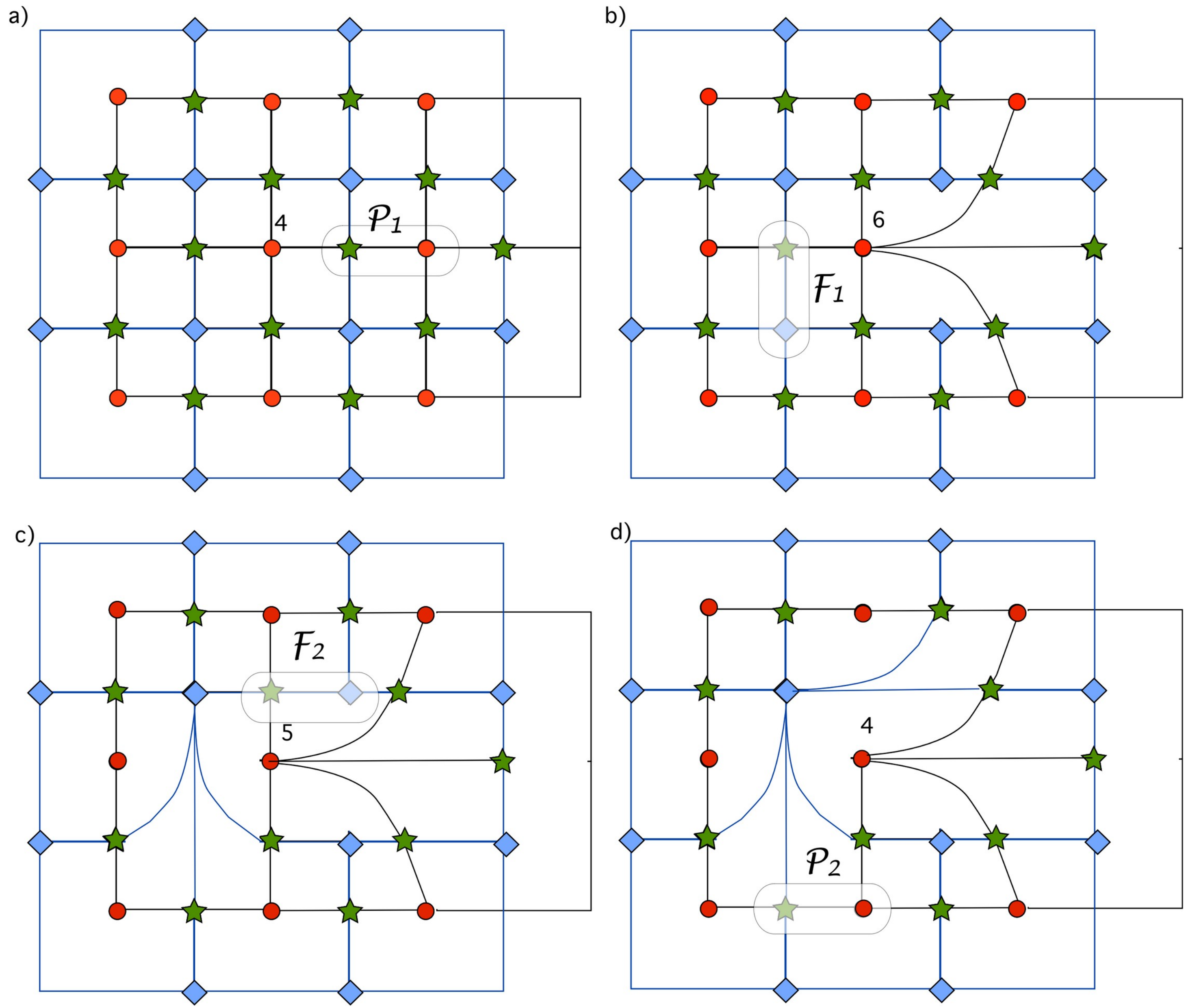


Constrained Excursion Set – quantitative results



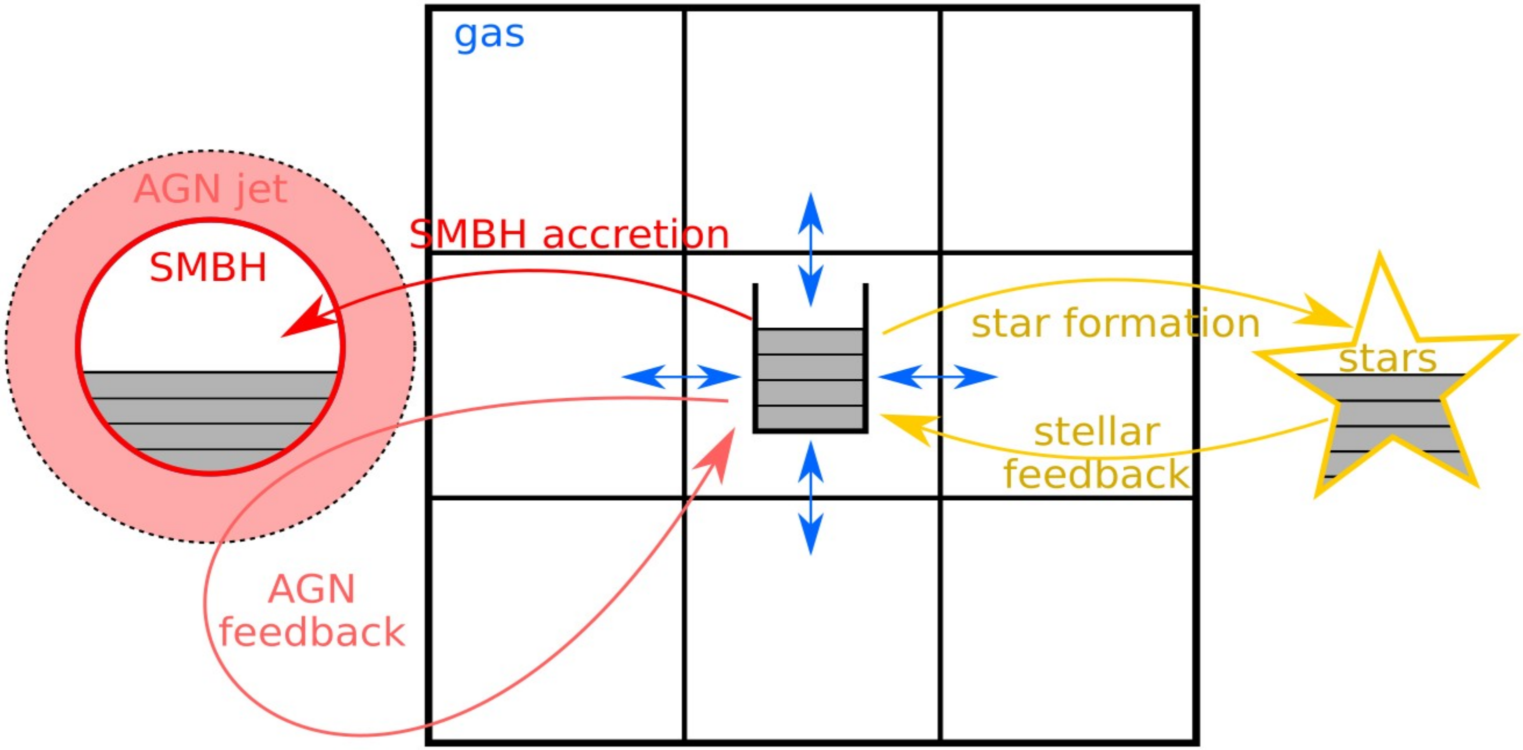
Typical mass (top), specific accretion rate (middle) and formation redshift (bottom) in the direction of the void (left) and the filament (top).

Connectivity and critical events – 2+1D case



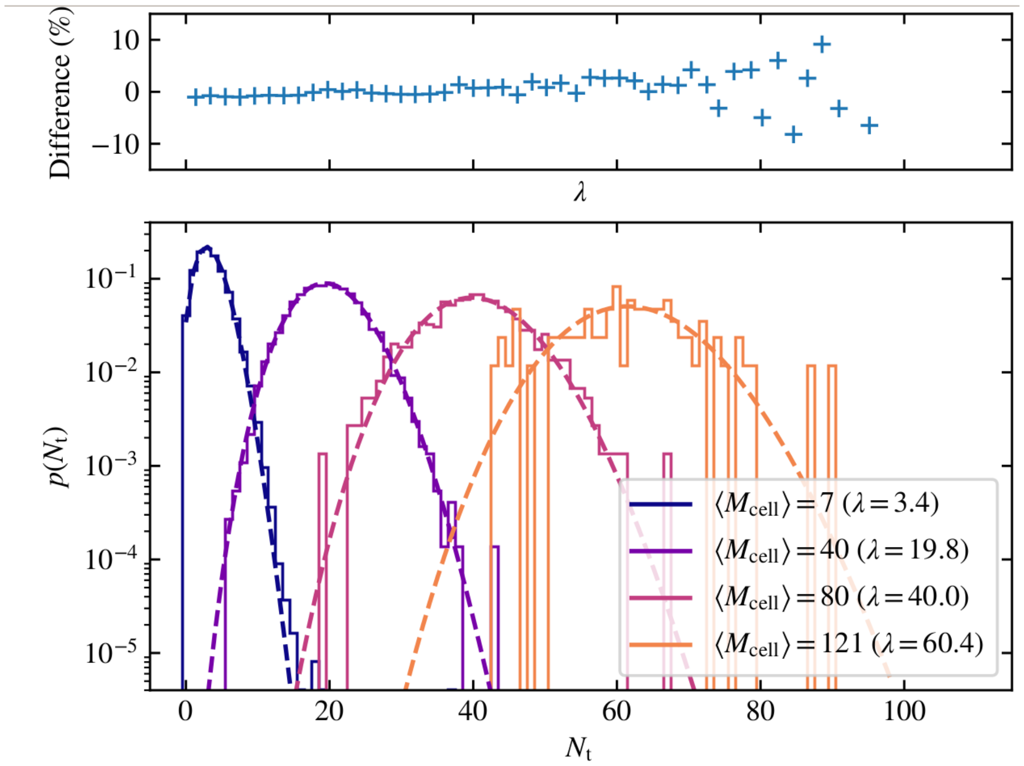
Typical evolution of the connectivity and corresponding critical points.

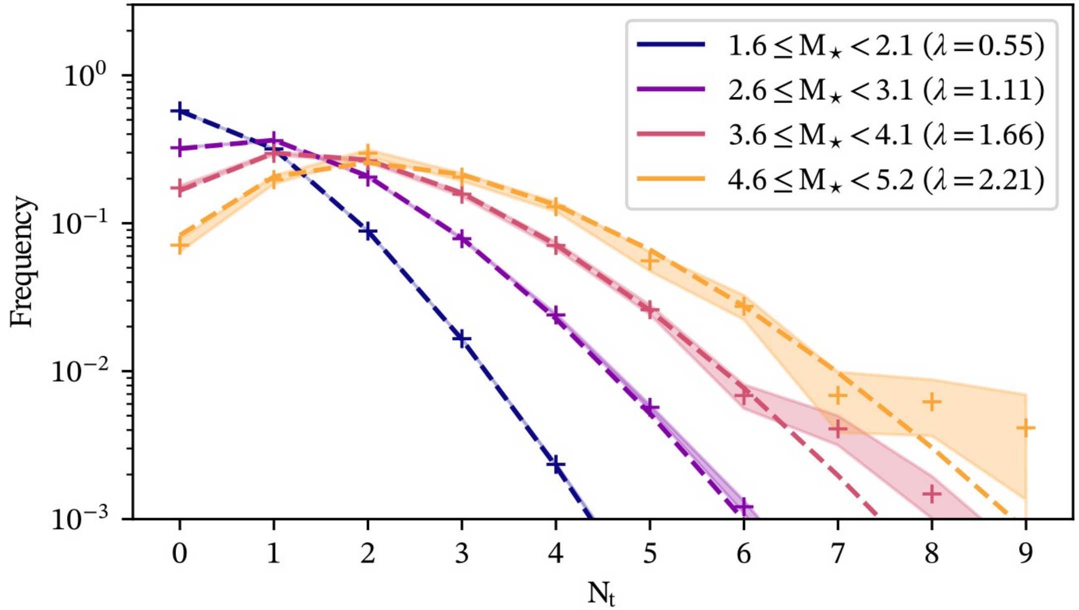
Monte Carlo tracers



$$p_{ij} = \frac{\Delta M_{ij}}{M_{ij}}$$

- M_{ij}:
  - Mass flux between cells
  - Newly-created star mass
  - Stellar feedback
  - Black hole accretion
- M:
  - Cell mass
  - Cell mass
  - Star mass
  - Cell mass



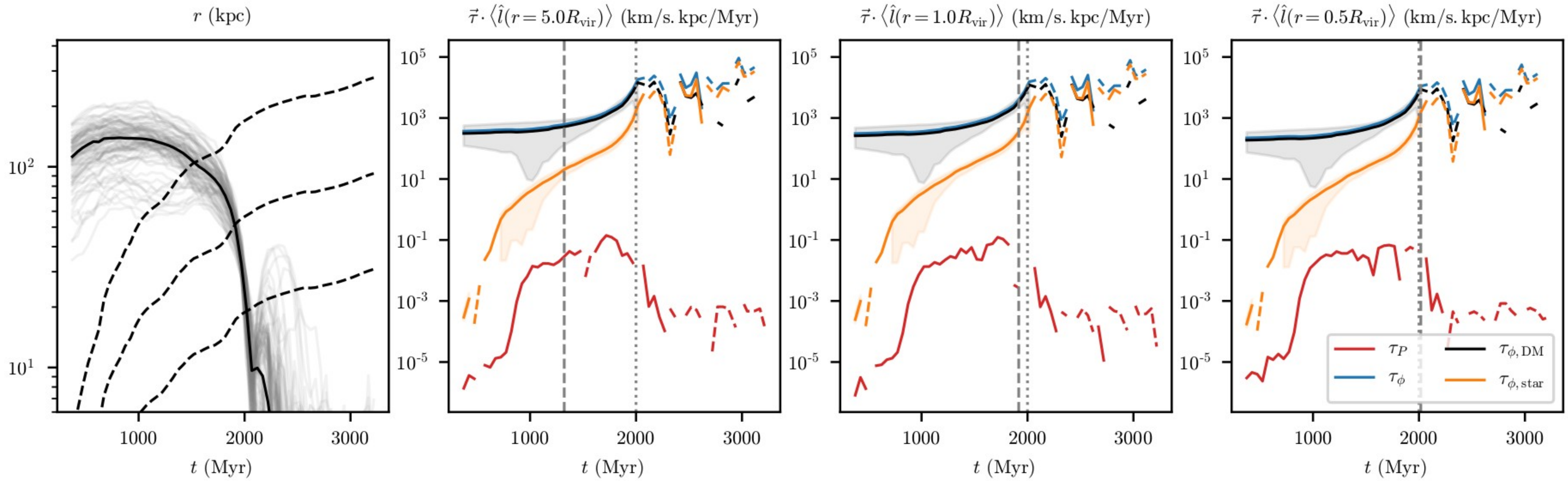


Gas tracer number density per **cell** mass bins

Star tracer particle number density per **star** mass bins

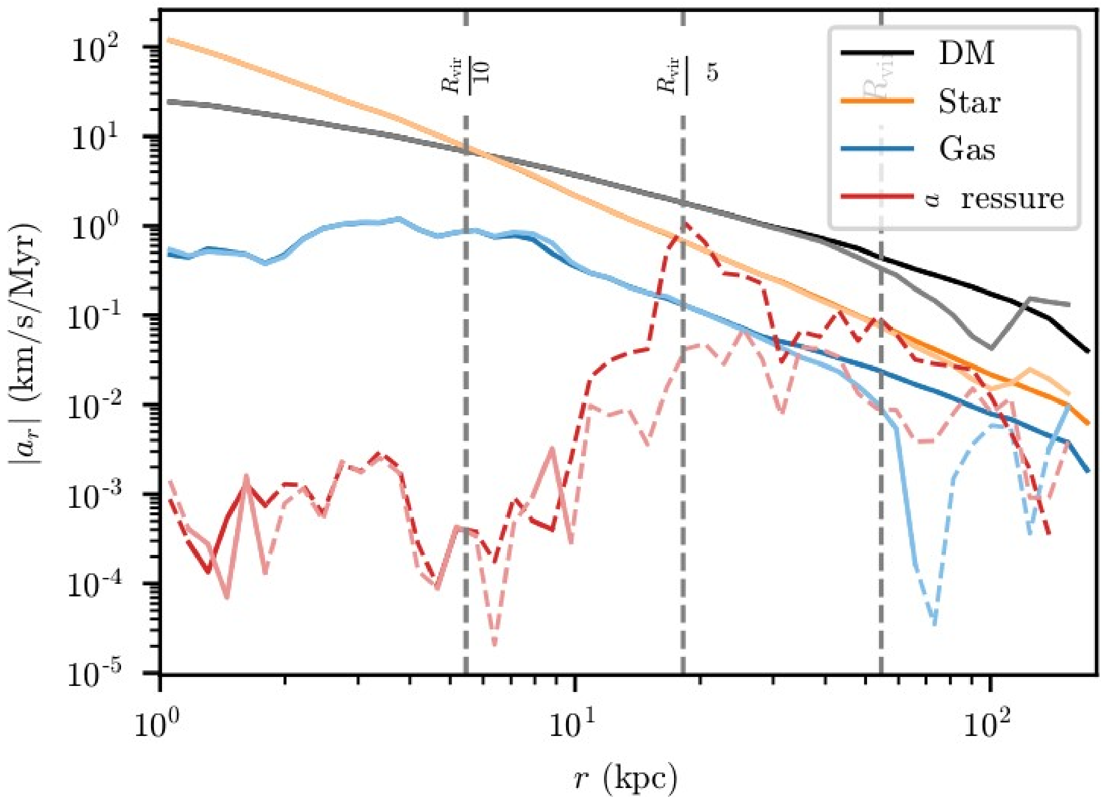
→ Number density consistent with Poisson distribution

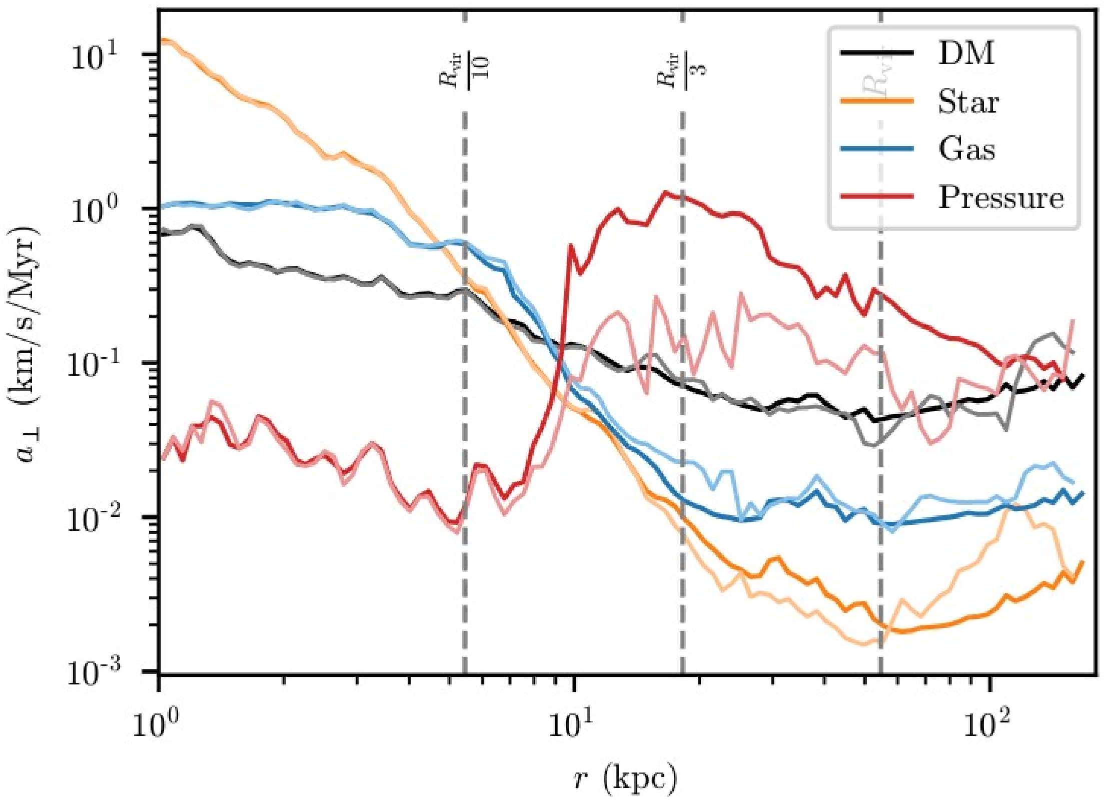
Torque along Lagrangian trajectory



Radius and mean torque magnitudes as a function of accretion time.

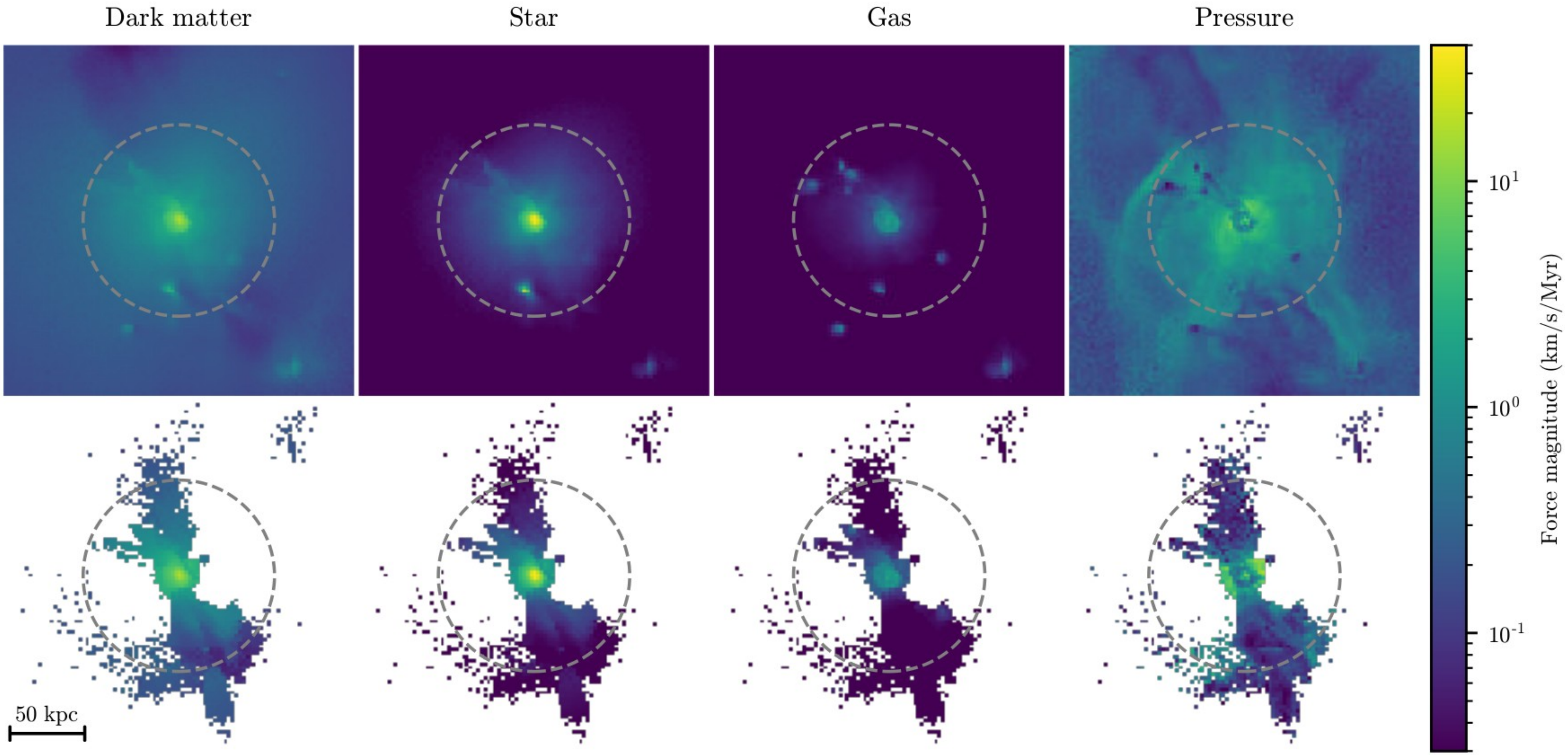
Acceleration profiles



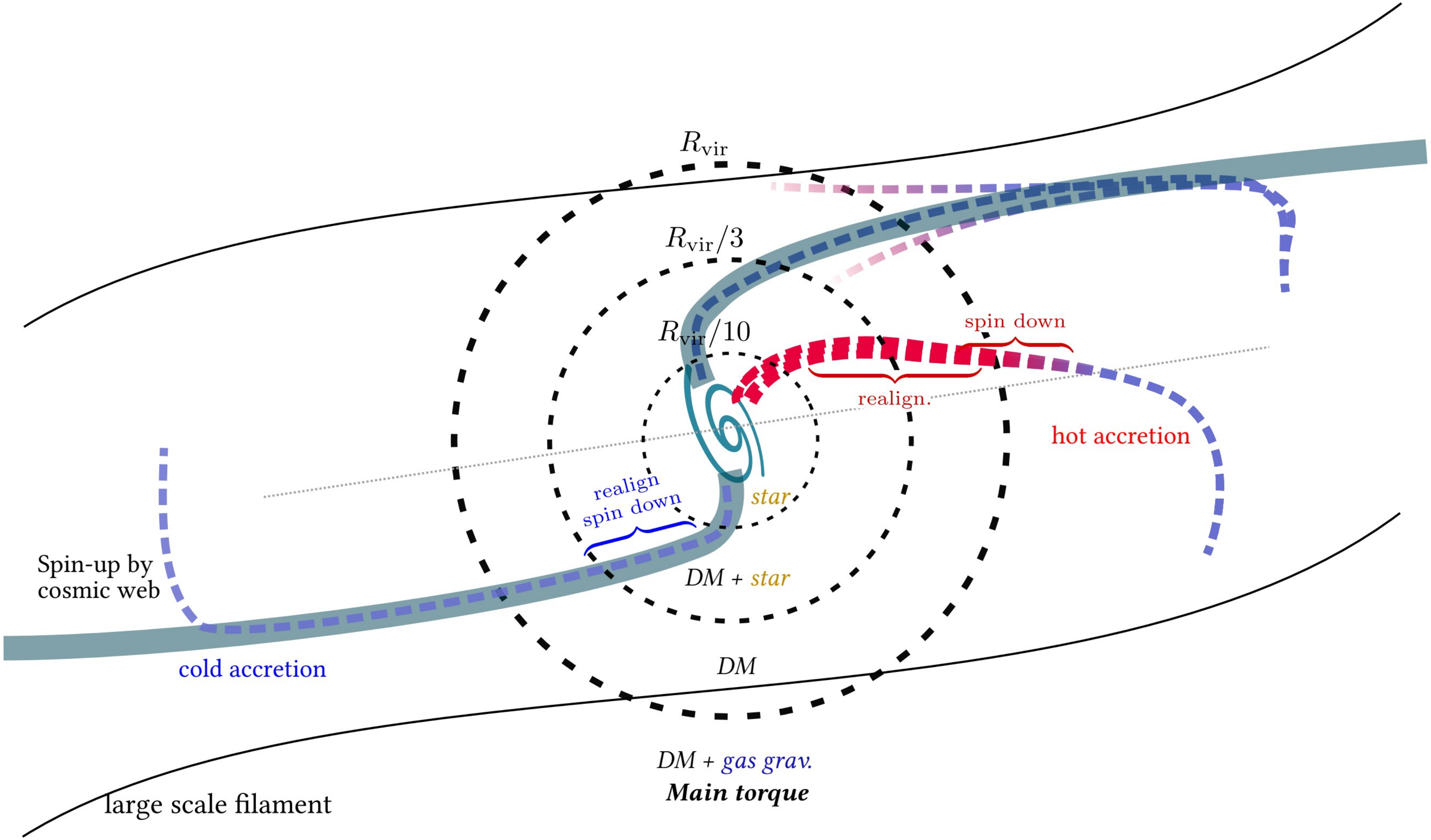


Acceleration profiles of one halo for the hot (dark) and cold-accreted (light) gas.

Acceleration profiles



Force projections around one halo for the hot (top) and cold-accreted (bottom) gas.



AM of cold gas

- **Amplitude** conserved down to **inner halo**
- **Alignment** ––––––––––––––––––––––––––––

AM of hot gas

- **Amplitude** conserved up to **virial shock**
- **Alignment** preserved down to **inner halo**